

TALAT Lecture 2401

Fatigue Behaviour and Analysis

81 pages, 56 figures

Advanced Level

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Objectives:

- To understand why, when and where fatigue problems may arise and the special significance to aluminium as structural material
- To understand the effects of material and loading parameters on fatigue
- To appreciate the statistical nature of fatigue and its importance in data analysis, evaluation and use
- To estimate fatigue life under service conditions of time-dependent, variable amplitude loading
- To estimate stresses acting in notches and welds with conceptual approaches other than nominal stress
- To provide qualitative and quantitative information on the classification of welded details and allow for more sophisticated design procedures

Prerequisites:

- Background in materials engineering, design and fatigue

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2401 Fatigue Behaviour and Analysis

Table of Contents

2401.01 Introduction.....	4
Significance of the Fatigue Problem and its Influence on Design	4
Significance of Fatigue for Aluminium Structures	5
Potential Sites for Fatigue Cracks.....	5
Conditions for Susceptibility	6
Definitions	9
Symbols.....	16
2401.02 Fatigue Damage and Influencing Parameters	17
Response of Material to Cyclic Loading.....	18
Generation of Fatigue Cracks	20
Fatigue Crack Growth.....	23
Crack Growth Mechanisms.....	26
Effect of other Parameters on Crack Propagation Rate	28
Endurance Limit.....	29
Predictive Theories of Fatigue	29
Damage Accumulation Theories.....	30
Manson-Coffin Law	30
Crack Growth Laws	32
Ideal Cumulative Damage Theory	32
2401.03 Fatigue Data Analysis and Evaluation.....	34
Analysis of Data.....	35
Analysis in the Middle-Cycle Fatigue Range	36
Analysis in the High-Cycle Fatigue Range	38
Fatigue Diagrams	39
Linear P-S-N Curves.....	41
Non-Linear P-S-N Curves.....	42
Some Problems of Data Analysis in Practice.....	43

2401.04 Load Spectra and Damage Accumulation	46
Service Behaviour	46
Time Dependent Loads	47
Spectrum Definition and Cycle Counting.....	47
The Rain-Flow Cycle Counting Method.....	50
The Service Behaviour Fatigue Test	51
Analytical Life Estimation and Damage Accumulation	54
The Palmgren-Miner Linear Damage Accumulation Hypothesis.....	55
Service Behaviour Assessment	58
Literature.....	60
2401.05 Local Stress Concepts and Fatigue.....	61
Analytical Relationship between Strain and Fatigue Life.....	61
Notch Theory Concept	62
The Strain-Life Diagram	63
References.....	65
Remarks	65
2401.06 Effects of Weld Imperfections on Fatigue	66
Types of Imperfections	67
Influence of Imperfections on Static Strength	69
Influence of Imperfections on Fatigue Strength.....	71
<i>Cracks</i>	72
<i>Porosity</i>	72
<i>Inclusions, Oxides</i>	74
<i>Lack of Penetration - Lack of Fusion</i>	74
<i>Weld Shape</i>	76
<i>Geometric Misalignment</i>	76
<i>Arc Strike, Spatter</i>	77
<i>Post-Weld Mechanical Imperfections</i>	77
2401.07 Literature/References	79
2401.08 List of Figures.....	80

2401.01 Introduction

- Significance of the fatigue problem and its influence on design
- Significance of fatigue for aluminium structures
- Potential sites for fatigue cracks
- Conditions for susceptibility
- Definitions
- Symbols

Significance of the Fatigue Problem and its Influence on Design

Structures subjected to fluctuating service loads in sufficient numbers are liable to fail by fatigue. Although the number of structures that have failed by fatigue under service conditions is low the consequences can be costly in terms of human life and/or property damage. Consequences may be catastrophic especially when no appropriate inspection intervals are observed and fatigue damage can grow and accumulate during service life.

Today's understanding of fatigue mechanisms, the experimental data available, in many cases the manufacturing process of constructional components, and the analytical methods applied offer a high degree of sophistication in the design procedures. However, currently the most important task is educational. It must be granted that all aspects of the fatigue problem and of fracture control are not yet universally available during engineering education. The following chapters try to give an overall outline of mechanisms, influencing parameters, analytical methods, and suggestions for better design. Beyond this the engineer must deal with actual problems. Experience is the best teacher, and so calculation examples have been added to the theoretical information.

For structures under fatigue loads the degree of compliance with the static limit state criteria given in other sections of these design rules may not serve as any useful guide to the risk of fatigue failure.

It is necessary therefore to establish as early as possible the extent to which fatigue is likely to control the design. In doing this the following factors are important.

- a) an accurate prediction of the complete loading sequence throughout the design life should be available.
- b) the elastic response of the structure under these loads should be accurately assessed
- c) detail design, methods of manufacture and degree of quality control can have a major influence on fatigue strength, and should be defined more precisely than for statically controlled members. This can have a significant influence on design and construction cost.

Significance of Fatigue for Aluminium Structures

Aluminium due to its physical and mechanical properties often finds application in areas where the ratio of dead weight of the structure to the total weight is significantly lower than in structures with steel or concrete. This implies that structural applications will be governed by a different ratio of minimum to maximum stresses. Such applications may also be frequently found in areas of severe environmental exposure. Because of applications in transportation area there is also a pressure on the design engineer to choose a light-weight, but then damage-prone, structure.

A great deal can be accomplished in terms of complementary methods of analysis utilising recent developments in fracture mechanics and respective data for aluminium alloys and joints in aluminium structures. It is only but recently that such methods and data have been systematically documented and are now been made available to practical design.

It should be kept in mind that rules and design values stated for steel structures cannot in every case be assumed for or adapted to respective problems in aluminium.

Potential Sites for Fatigue Cracks

Most common initiation sites for fatigue cracks are as follows:

- a) toes and roots of fusion welds
- b) machined corners and drilled holes
- c) surfaces under high contact pressure (fretting)
- d) roots of fastener threads

In a typical structural component designed statically the fatigue assessment will not normally present a more severe demand to be fulfilled. Only in cases of respective applications with frequent and significant load variations fatigue will be the governing criterion.

In a comparative study (Kosteas/Ondra: "Abgrenzung der Festigkeitsnachweise im Leichtmetallbau. Research report 235/91, Munich 01.10.1992) for an aluminium column for which both a static and a fatigue assessment were performed, it can be demonstrated that the static limit state assessment (ultimate limit state, flexural/torsional buckling, local buckling, deflection) limits the applicability range of the fatigue design S-N curves to approx. above 10^5 cycles. An extrapolation of design S-N curves down to 10^4 will rarely have any practical meaning. Bearing in mind that with a variation or enhancement of the geometrical dimensions of the calculated case the relative static limit state assessments will have a more pronounced influence, the applicability range of fatigue design moves to even higher cycle numbers.

Conditions for Susceptibility

The main conditions affecting fatigue performance are as follows:

- a) *high ratio of dynamic to static load.* Moving or lifting structures, such as land or sea transport vehicles, cranes, etc. are more likely to be prone to fatigue problems than fixed structures unless the latter are predominately carrying moving loads as in the case of bridges.
- b) *frequent applications of load.* This results in a high number of cycles in the design life. Slender structures or members with low natural frequencies are particularly prone to resonance and enhance magnification of dynamic stress, even though the static design stresses are low. Structures subjected predominantly to fluid loading, such as wind and structures supporting machinery, should be carefully checked for resonance effects.
- c) *use of welding.* Some commonly used welded details have low fatigue strength. This applies not only to joints between members, but also to any attachment to a loaded member, whether or not the resulting connection is considered to be 'structural'.
- d) *complexity of joint detail.* Complex joints frequently result in high stress concentrations due to local variations in stiffness of the load path. While these may have little effect on the ultimate static capacity of the joint, they can have a severe effect on fatigue resistance. If fatigue is dominant, the member cross-sectional shape should be selected to ensure smoothness and simplicity of joint design, so that stresses can be calculated and adequate standards of fabrication and inspection can be assured.
- e) *environment.* In certain thermal and chemical environments fatigue strength may be reduced.

Figure 2401.01.01 Fatigue - Where?

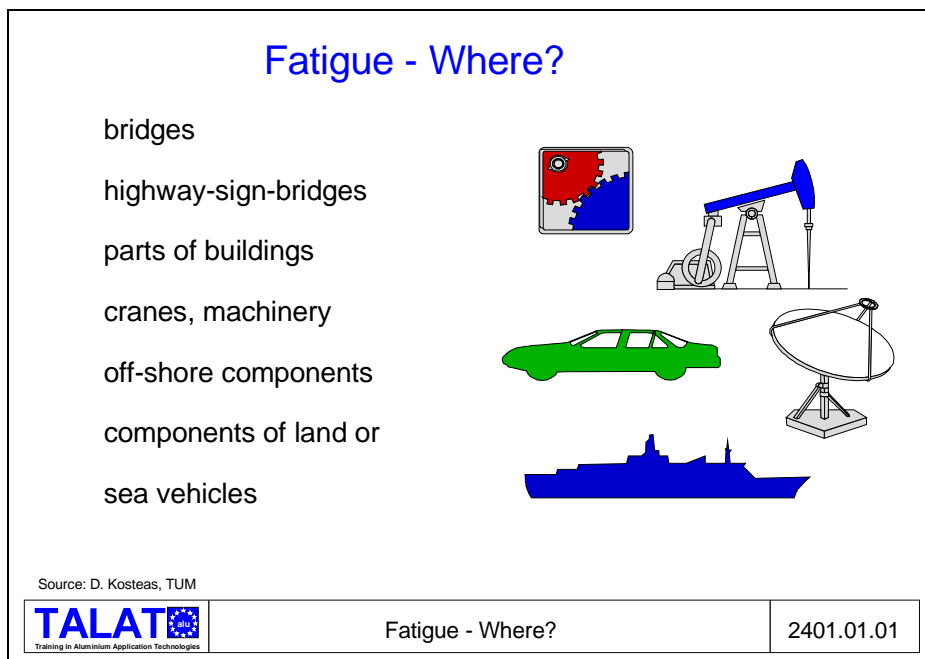


Figure 2401.01.02 Fatigue - Location

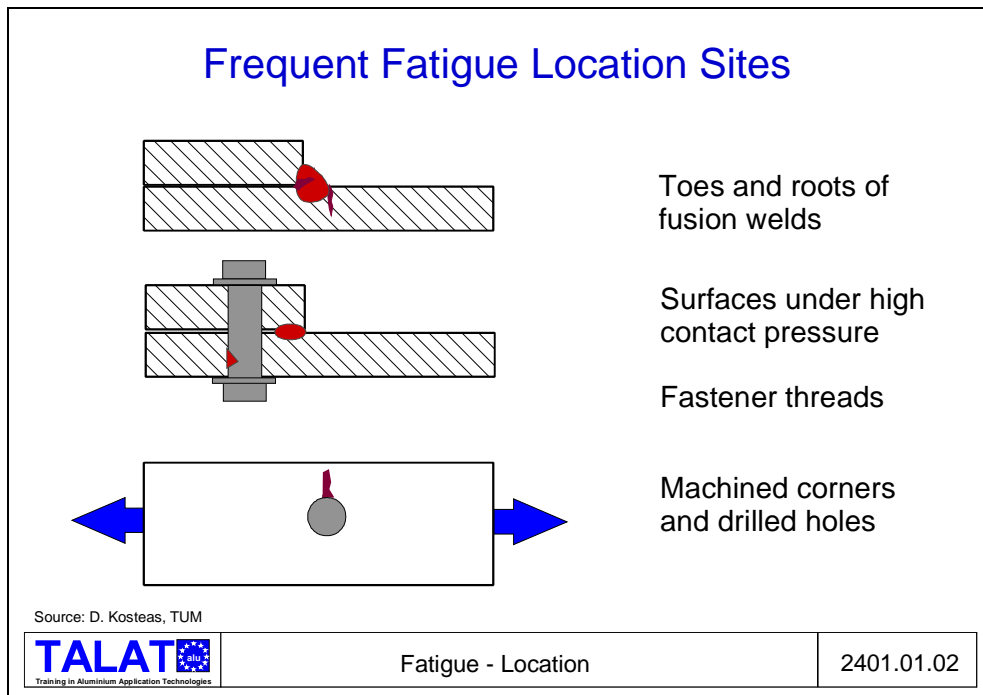


Figure 2401.01.03 Fatigue - When and What?

Fatigue - When and What?

When: subjected to fluctuating service loads in sufficient numbers

What: cracks initiate + propagate to final failure

consequences are costly in terms of human life and property damage

Source: D. Kosteas, TUM


	Fatigue - When and What?	2401.01.03
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Figure 2401.01.04 Fatigue - The Remedy






<h2 style="color: blue;">Fatigue - The Remedy</h2>		
	educate for good design + fracture control	
	predict service load history, assess damage	
	Bear in mind:	
	<input type="checkbox"/>	design,
	<input type="checkbox"/>	manufacturing and
	<input type="checkbox"/>	degree of quality control have major influence on fatigue strength, and influence
	<input type="checkbox"/>	design and construction costs significantly
Source: D. Kosteas, TUM		
	Fatigue - The Remedy	2401.01.04

Figure 2401.01.05 Fatigue - Significance for Aluminium

<h2 style="color: blue;">Fatigue - Significance for Al</h2>		
<p>Due to its physical and mechanical properties (low specific weight, low modulus of elasticity, corrosion resistance) aluminium is found frequently in applications with:</p>		
<input type="checkbox"/>	ratio $\sigma_{\min}/\sigma_{\max}$ different from steel or concrete	
<input type="checkbox"/>	severe environmental exposure	
<input type="checkbox"/>	light-weight transportation,	 damage-prone
Source: D. Kosteas, TUM		
	Fatigue - Significance for Al	2401.01.05

Definitions

Block	a specified number of constant amplitude loading cycles applied consecutively, or a spectrum loading sequence of finite length that is repeated identically
Confidence Interval	an interval estimate of a population parameter computed so that the statement 'the population parameter included in this interval will be true, on the average, in a stated portion of the times such computations are made based on different samples from the population.
Confidence Level	the stated proportion of the times the confidence interval is expected to include the population parameter
Confidence Limits	the two statistics that define a confidence interval
Constant Amplitude Loading	a loading in which all of the peak loads are equal and all of the valley loads are equal
Constant Life Diagram	a plot (usually on rectangular co-ordinates) of a family of curves each of which is for a single fatigue life, N , relating stress amplitude $\Delta\sigma$, to mean stress σ_m or maximum stress σ_{max} , or to minimum stress, σ_{min} .
Corrosion Fatigue	synergistic effect of fatigue and aggressive environment acting simultaneously, which leads to a degradation in fatigue behaviour.
Counting Method	a method of counting the occurrences and defining the magnitude of various loading parameters from a load-time history; (some of the counting methods are: level crossing count, peak count, mean crossing peak count, range count, range-pair count, rain-flow count, race-track count).
Crack Size a	a lineal measure of a principal planar dimension of a crack, commonly used in the calculation of quantities descriptive of the stress and displacement fields, and often also termed 'crack length'.
Cut-Off Limit	the fatigue strength at $1 \cdot 10^8$ cycles corresponding to the S-N curve. All stresses below this limit may be ignored

Cycle	one complete sequence of values of load that is repeated under constant amplitude loading. The symbol n or N is used to indicate the number of cycles (see definition of fatigue life).
Cycles Endured	the number of cycles of specified character (that produce fluctuating load) which a specimen has endured at any time in its load history
Cycle Ratio	the ratio of cycles endured, n , to the estimated fatigue life (S-N) or the strain versus fatigue life (ϵ -N) diagram for cycles of the same character, that is, $C=n/N$
Design Life	the period during which the structure is required to perform without repair
Design Spectrum	the total of all stress spectra, caused by all loading events during design life, to be used in the fatigue assessment
Discontinuity	an absence of material causing stress concentration. Typical discontinuities are cracks, scratches, corrosion pits, lack of penetration, porosity or undercut
Environment	the aggregate of chemical species and energy that surrounds the test specimen
Estimation	a procedure for making a statistical inference about the numerical values of one or more unknown population parameters from observed values in sample
Fail-Safe	fatigue limit state assessing the gradual, stable crack propagation
Fatigue	the process of progressive localised permanent structural change occurring in a material subjected to conditions that produce fluctuating stresses and strains at some point or points and that may culminate in cracks or complete fracture after a sufficient number of fluctuations
Fatigue Crack Growth Rate	the rate of crack extension caused by constant amplitude fatigue loading, expressed in terms of crack extension per cycle of fatigue (da/dN)
Fatigue Life N	the number of loading cycles of a specified character that a given specimen sustains before failure of a specified nature occurs

Fatigue Limit σ_f	theoretically the fatigue strength for $N \rightarrow \infty$, or the limiting value of the median fatigue strength as the fatigue life, N , becomes very large. When all stresses are less than the fatigue limit, no fatigue assessment is required. In most cases the fatigue limit is given at $2 \cdot 10^6$ or $5 \cdot 10^6$ cycles.
Fatigue Loading	periodic or non-periodic fluctuation loading applied to a test specimen or experienced by a structure in service (also known as cyclic loading).
Fatigue Notch Factor K_f	the ratio of the fatigue strength of a specimen with no stress concentration to the fatigue strength of a specimen with a stress concentration for the same percent survival at N cycles and for the same conditions
Fatigue Notch Sensitivity	a measure of the degree of agreement between fatigue notch factor K_f and the theoretical stress concentration factor K_t
Fatigue Strength	a value of stress for failure at exactly N cycles as determined from a S-N diagram. The value of S_N thus determined is subject to the same conditions as those which applied to the S-N diagram
Group	the specimens of the same type tested at one time, or consecutively, at one stress level. A group may comprise one or more specimens
Hysteresis	the stress-strain path during the cycle
Loading Amplitude	one half of the range of a cycle
Loading Event	a well defined loading sequence on the entire structure caused by the occurrence of a loading. This may be the approach, passage and departure of a vehicle or device on the structure
Loading (Unloading) Rate	the time rate of change in the monotonic increasing (decreasing) portion of the load time function
Load Stress Ratio R	the algebraic ratio of the two loading parameters of a cycle; the most widely used ratio is $R = P_{\min}/P_{\max} = \sigma_{\min}/\sigma_{\max}$
Maximum Load P_{\max}	the load having the highest algebraic value

Maximum Stress σ_{\max}	the stress having the highest algebraic value
Mean Load P_m	the algebraic average of the maximum and minimum loads in constant amplitude loading, or of individual cycles in spectrum loading, or the integral average of the instantaneous load values of a spectrum loading history
Mean Stress σ_m	the algebraic average of the maximum and minimum stresses in constant amplitude loading, or of individual cycles in spectrum loading, or the integral average of the instantaneous stress values of a spectrum loading history
Miner's Summation	a linear damage accumulation calculation (Palmgren-Miner rule)
Minimum Load P_{\min}	load having the lowest algebraic value
Minimum Stress σ_{\min}	stress having the lowest algebraic value
Nominal Stress	the applied stress calculated on the area of the net section of the structural component by simple theory ignoring stress raisers and disregarding plastic flow
Parameter	a constant (usually to be estimated) defining some property of the frequency distribution of the population, such as a population median or a population of a standard deviation
Peak	the occurrence where the fatigue derivative of the load time history changes from positive to negative sign; the point of maximum load in constant amplitude loading
Population	the totality of set of test specimens, real or conceptual, that could prepared in the specified way from the material under consideration
Probability of Failure P	the ratio of the number of observations failing in fatigue to the total number of observations
Probability of Survival Q	it follows that $P + Q = 1$
Random Loading	a spectrum load where the peak and valley loads and their sequence result of a random process; the loading is usually described in terms of its statistical properties, such as the probability density function,

	the mean, the root mean square, the irregularity factor, and others as appropriate
Range ΔP , $\Delta\sigma$, $\Delta\varepsilon$	the algebraic difference between successive valley and peak loads (positive range or increasing load range) or between successive peak and valley loads (negative range or decreasing load range).
Reversal	the occurrence where the first derivative of the load-time history changes sign
Run-Out	the piece at a number of cycles at which no apparent fatigue damage has been observed and test is continued
Sample	the specimens selected from the population for test purposes
Significance	an effect or difference between populations is said to be present if the value of a test-statistic is significant, that is, lies outside of selected limits
Survival Limit p%	a curve fitted to the fatigue life for p% survival values of each several stress levels. It is an estimate of the relationship between stress and the number of cycles to failure that p% of the population would survive, p may be any percent (in most cases p is set equal 97.5%)
S-N Diagram	a plot of stresses against the number of cycles to failure. The stress can be the maximum stress σ_{\max} , the minimum stress σ_{\min} , or stress range $\Delta\sigma$. The diagram indicates the S-N relationship for a specified value of σ_m or R and a specified probability of survival. For N a log scale is almost always used. For σ a linear scale or log scale is used in most cases.
Spectrum Loading	a loading in which all of the peak loads are not equal or all of the valley loads are not equal (also known as variable amplitude loading)
Statistic	a summary value calculated from the observed values in a sample
Stress Level	the pair of stress (or strain) components necessary to define the applied cycle

Stress Ratio R	the algebraic ratio of the minimum stress to the maximum stress in one cycle, $R = \sigma_{\min}/\sigma_{\max}$
Stress Intensity Factor K	the magnitude of the ideal-crack-tip stress field (a stress-field singularity) for a particular mode in a homogeneous, linear elastic body
Stress Concentration Factor K_t	the ratio of the greatest stress in the region of a notch as determined by advanced theory to the corresponding stress
Test of Significance	a statistical test that purports to provide a test of a null hypothesis, for example, that an imposed treatment in the experiment is without effect
Tolerance Interval	an interval computed so that it will include at least a stated percentage of the population with stated probability
Tolerance Limit	the two statistics that define a tolerance interval
Truncation	the exclusion of cycles with values above, or the exclusion of cycles with values below a specified level (referred to as truncation level) of a loading parameter
Valley	The occurrence where the first derivative of the load time history changes from negative to positive sign; the point of minimum load in constant amplitude loading
Variable Amplitude Loading	see spectrum loading

Figure 2401.01.06 Definitions: Constant Amplitude Loading

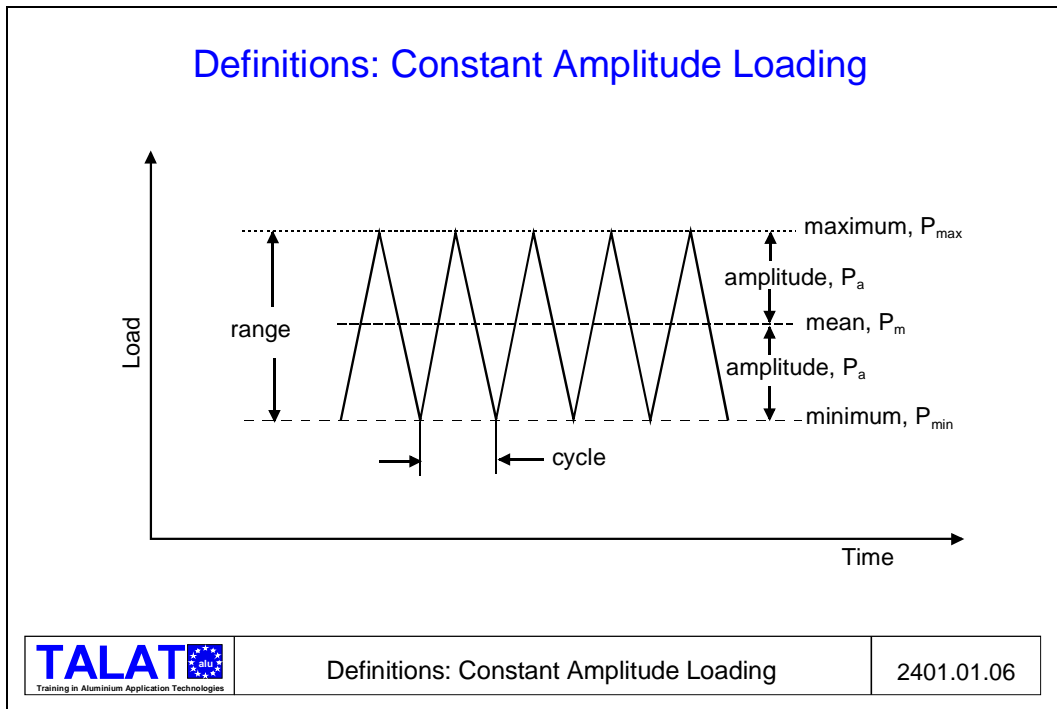
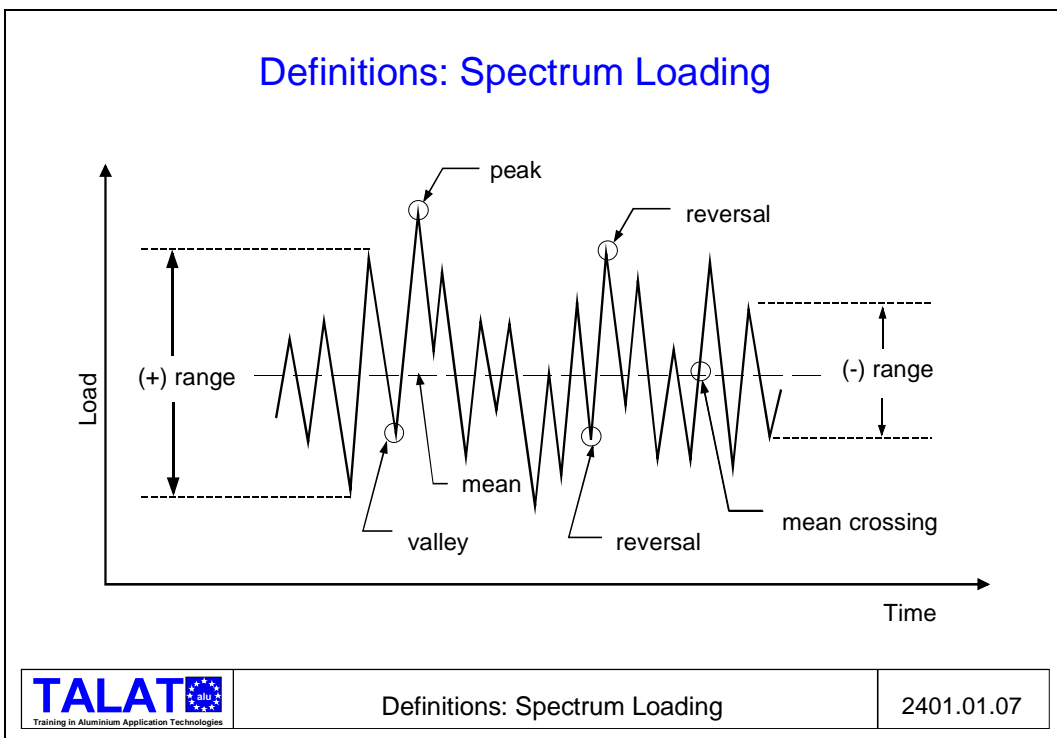


Figure 2401.01.07 Definitions: Spectrum Loading



Symbols

a	crack length
e	eccentricity
E	modulus of elasticity
G	shear modulus
K	stress intensity factor
l	length of attachments
m	slope constant in fatigue strength equation
n_i	number of cycles corresponding to specified stresses σ_i
N	number of cycles corresponding to a particular fatigue strength
N_c	cut-off-limit
R	stress ratio $R = \sigma_{\min}/\sigma_{\max}$
t	plate thickness
R_m	ultimate tensile strength
$R_{p,0.2}$	tensile strength at $\epsilon = 0.2\%$
ϵ_{\max}	maximum strain
ϵ_{\min}	minimum strain
$\Delta\epsilon$	strain range
σ_{\max}	maximum stress
σ_{\min}	minimum stress
σ_{res}	residual stress
$\Delta\sigma$	stress range $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$
$\Delta\sigma_e$	equivalent constant amplitude stress calculated from the respective stress range spectrum for a particular value for m
τ	shear stress

2401.02 Fatigue Damage and Influencing Parameters

- Response of material to cyclic loading
- Generation of fatigue cracks
- Fatigue crack growth
- Crack growth mechanism
- Effect of other parameters on crack propagation rate
- Endurance limit
- Predictive theories of fatigue
- Damage accumulation theories
- Manson-Coffin law
- Crack growth laws
- Ideal cumulative damage theory

Fatigue damage occurs in metals due to local concentrations of plastic strain. Consequently minimising these strain concentrations must be the first rule for avoiding fatigue failure. As an alternative or complementary method a material must be chosen which best resists the mechanisms which lead to cracks and their growth.

It is important to understand these mechanisms and the influence of them on material properties, load amplitudes and their sequence, temperature and environment. We treat the problem in a simplified view recognising the fact that real structures contain discontinuities which may develop into cracks with applications of stress, progressive crack extension following up to final failure. We study thus the material response to cyclic loading at temperatures in the sub-creep range. Procedures for minimising, repairing and detecting fatigue damage can be thus applied intelligently.

By far the most common mode of fatigue failure consists simply of initiation and propagation of cracks to the point of static failure. A degradation of material properties during fatigue is not normally observed.

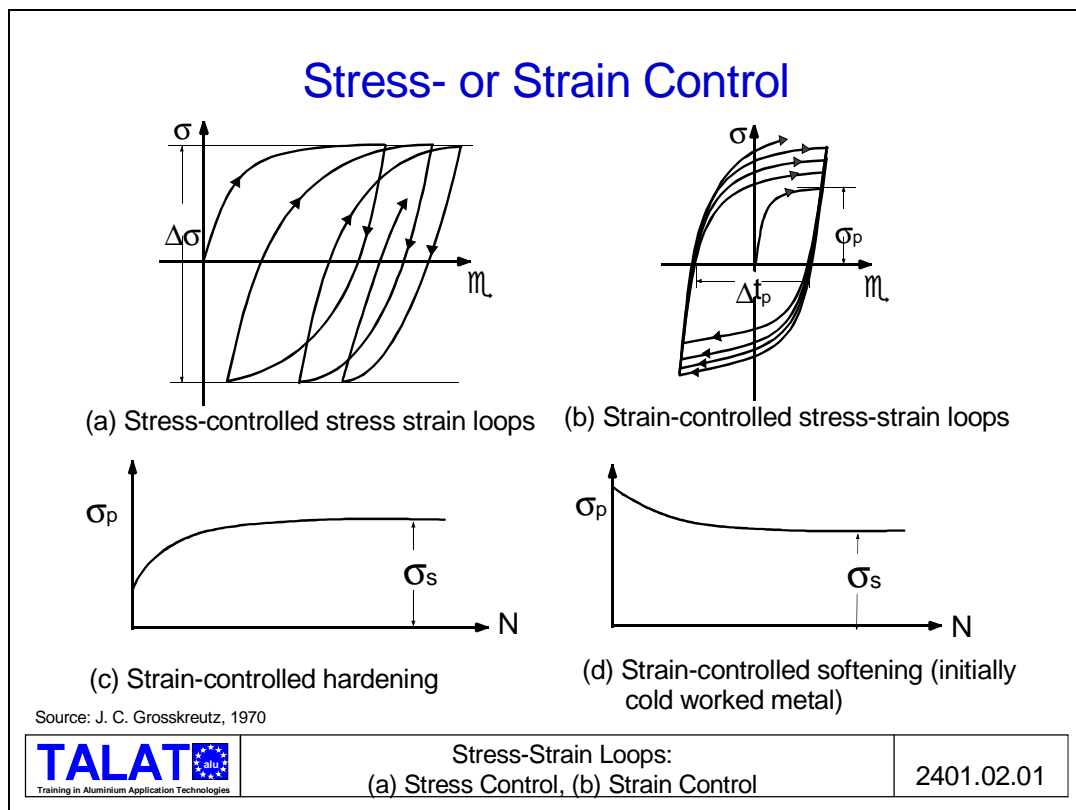
Superimposed on the general response of the material to cyclic loading, i.e. hardening or softening depending on the material, the load amplitude and the temperature, localised plastic deformation develops at stress concentration points. This repeated, localised plastic deformation leads to crack initiation. Provided the local stress concentration exceeds a certain threshold, a fatigue crack, once initiated, continues to grow a finite amount during each cycle.

Crack propagation occupies a major portion of fatigue life, especially at high load amplitudes. Final failure may be the result of the growth of one crack, or of many small cracks coalescing into a final crack. The higher the load amplitude, the more likely the production of multiple cracks. Corrosive environment also produces multiple cracking and accelerates failure.

Final catastrophic failure occurs when a crack has grown to a critical length such that the next application of load produces static failure of the remaining net section. Under service conditions of variable amplitude loading, the critical crack length must be defined of course, in terms of the highest expected load and the total load spectrum. Redundant structures, i.e. structures designed with 'crack arrestors' stopping the rapid growth of critical cracks before they weaken the integrity of the total structure, represent an important advance in fatigue design based on the previously described crack growth mechanisms.

Response of Material to Cyclic Loading

The familiar stress-strain relation and work hardening under static, tensile loading condition forms practically the first quarter-cycle of an extended fatigue test. The fatigue test itself can be performed in a stress- or strain-controlled manner. The stress amplitude σ or strain amplitude ϵ , the mean stress or strain, and the number of cycles N characterise the usual fatigue test. For strain-controlled tests one has to distinguish between control of either the total or the plastic strain amplitude.



If the stress range $\Delta\sigma$ is controlled and maintained constant, the strain amplitude gradually decreases, **Figure 2401.02.01**. If the plastic strain range $\Delta\epsilon_p$ is controlled, the stress amplitude required to maintain the strain limit gradually increases. A fatigue hardening or softening curve is represented by the peak stress amplitude σ_p when each cycle is plotted against the number of cycles. Such curves generally show that rapid hardening or softening occurs in the first few percent of total fatigue life. Eventually the material 'shakes down' into a steady-state or saturation condition in which the rate of hardening or softening becomes zero. The magnitude of the saturation stress, σ_s depends on the

plastic strain amplitude, the temperature and the initial degree of cold work - σ_s increases when $\Delta\varepsilon_p$ is increased or the temperature is decreased. Aluminium exhibits generally a unique saturation stress σ_s for a given $\Delta\varepsilon_p$ and temperature independent of prior load history (the so-called wavy-slip mode).

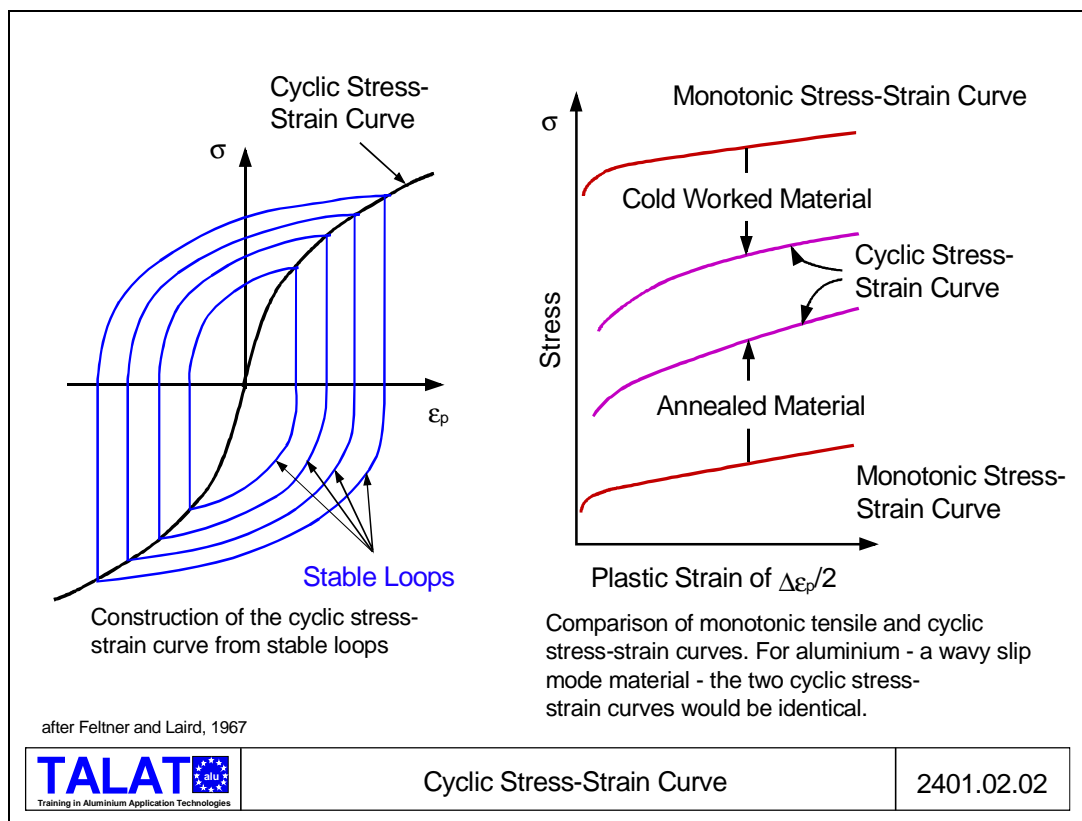
The locus of the tips of all steady-state cyclic loops of width $\Delta\varepsilon_p$ forms the so-called 'cyclic stress-strain curve', i.e. the steady-state cyclic stress strain behaviour of the material. The curve is generated by plotting (for a given temperature and initial condition) the saturation stress vs. $\Delta\varepsilon_p/2$, (**Figure 2401.02.02**). An approximate expression for the cyclic stress-strain curve is:

$$\sigma_s = \sigma_0 (\Delta\varepsilon_p/2)^{n'}$$

where

σ_0 is a constant and n' is the cyclic strain hardening exponent with a value of approximately 0.15 for wavy-slip mode materials such as aluminium.

The above equation is extremely useful in low cycle fatigue applications. It gives in convenient form the stable stress amplitudes to be expected from a given imposed strain amplitude, or vice-versa. Using it, one can convert easily from Manson-Coffin type plots of $\log(\Delta\sigma_p)$ vs. $\log(n)$ to S-N plots.



On a microscopic scale the movement of dislocations in the crystal structure leads to the saturation condition as described above and serves only to determine the flow stress of the metal, i.e. its hardness.

Fatigue hardening/softening mechanisms in complex, precipitation hardened alloys such as the 7000 series of aluminium alloys are not as straightforward. Nevertheless, the main product of the cycling is again a dense array of dislocations whose presence per se does not serve to weaken the material. In certain, specific cases, cyclic straining can cause actual degradation of the precipitate structure in an alloy, thus causing irreversible softening. The softened zones can then become the sites of fatigue failure. A controversy exists over whether such action occurs in high-purity Al-4Cu alloys. No such softening has ever been observed in commercial aluminium alloys, however (**Figure 2402.02.02**).

In the presence of stress concentration, enhanced fatigue hardening will occur in proportion to the stress or strain concentration factor. One can expect this enhancement in the vicinity of notches, fasteners, welds, and most importantly, near the tip of a fatigue crack.

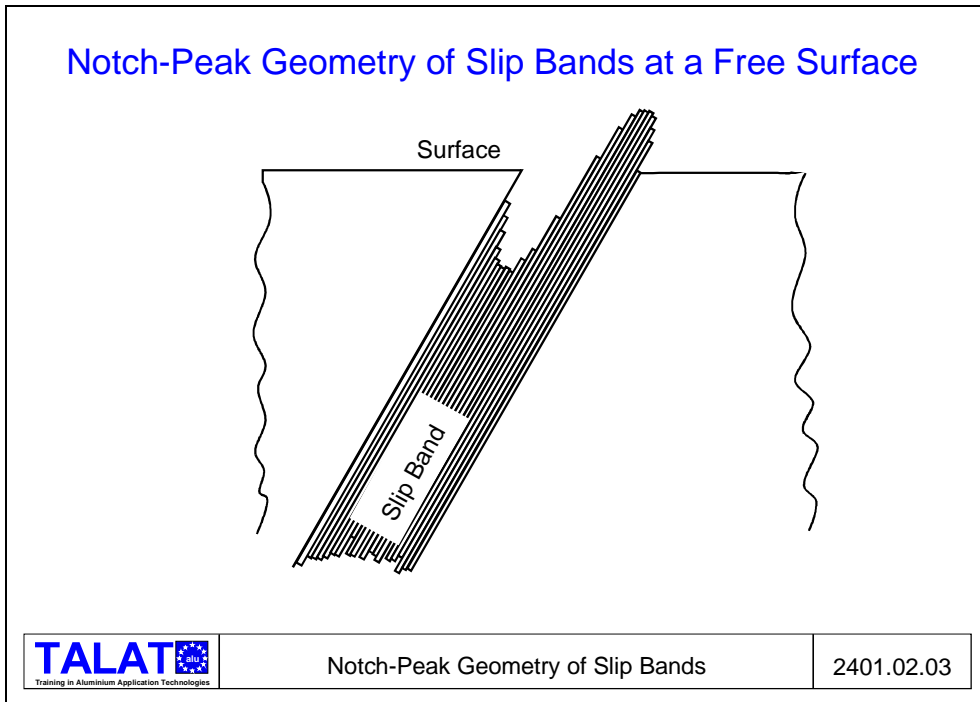
Generation of Fatigue Cracks

When is a crack ?

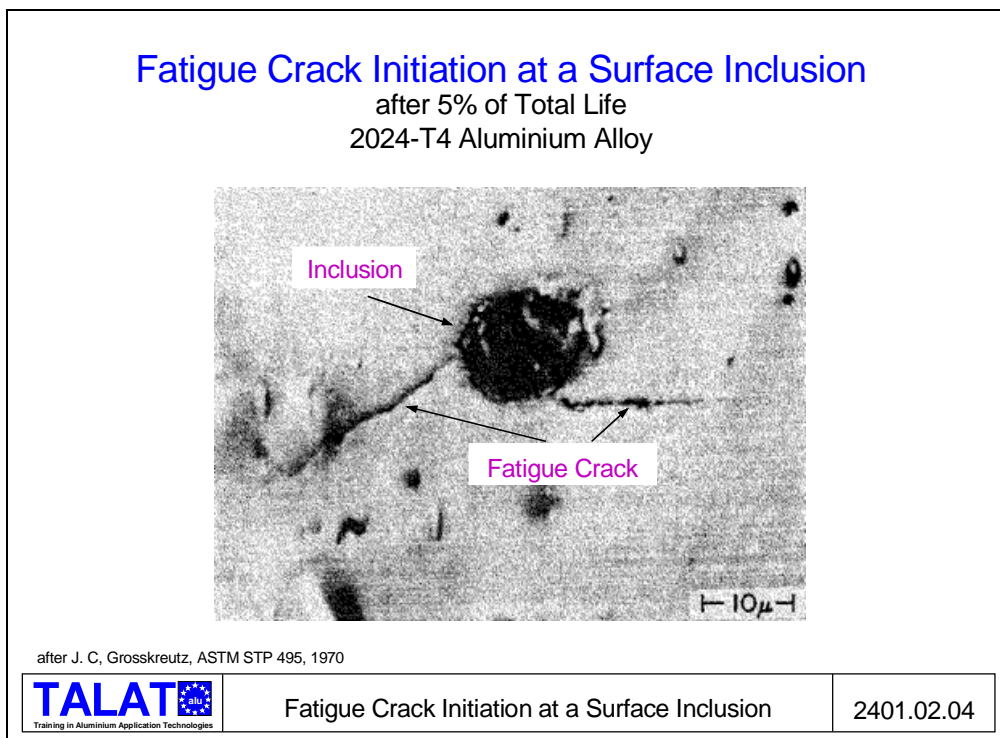
For the practitioner a crack exists when he can see it with the observational technique, he normally employs for such purposes, the naked eye, a glass magnifier, a metallographic microscope, or an electron microscope. A fatigue crack may vary in length from anywhere from 3mm down to 1000Å.

Defining a crack in terms of the highest resolving power instrument available (the electron microscope for instance) it is possible to establish a number of load cycles N_I , to generate an observable fatigue crack. It is usual to express the result in terms of the fraction of total life, N_I/N_T . It has been shown that this ratio is normally a small number in unnotched members, about 0.1, so that fatigue crack propagation occupies a large percentage of total life. This is especially true in structural components with existing imperfections due to manufacturing. Nevertheless the mechanisms of crack initiation and respective estimation of life cycles are important for certain applications in the high-cycle range.

As already mentioned fatigue cracks always begin at concentrations of plastic strain. Consequently if no other manufacturing imperfections are present fatigue cracks have their origins at the surface. The so-called slip band formation, extrusions and intrusions on the surface of an otherwise uncracked material form fatigue crack initiation sites (**Figure 2401.02.03**).



Under high amplitude loading fatigue cracks start at grain boundaries in pure aluminium. In many commercial alloys the existence of large second phase particles, inclusions, play a predominant role in crack generation. They cause localised plastic deformation leading to cracking usually at the inclusion matrix interface (**Figure 2401.02.04**).



Other flaws such as internal voids or large surface scratches may be the sites of fatigue crack generation. Such flaws need only the application of cyclic load to begin their growth as bona fide fatigue cracks.

Joints are an important site for fatigue crack generation in structures since they exhibit substantial stress concentration points, notches, either on the free surface of the component or at internal surfaces, such as lack of penetration in welds. Another cause for fatigue crack formation at joints may be fretting.

There is indication that cracks must reach a minimum critical size before they can begin propagating. Nonpropagating microcracks which have been observed at very low stress amplitudes may be examples of cracks, for which the generation mechanism ceased to operate before they reached critical size.

The value of N_I/N_T is dependent of the load amplitude, specimen geometry, material properties, the temperature, previous loading history, and the environment. In very general terms the following statements help to describe the various effects.

The value of N_I/N_T decreases with increasing load amplitude, so that in the extreme low cycle range the entire life is consumed in crack propagation and in the extreme high cycle range a substantial portion of the entire life is consumed in crack initiation. With increasing load amplitude a larger number of cracks are generated. A stress concentration will reduce N_I/N_T (Table 1). Microcracks will develop more quickly in wavy-slip mode materials. Cracks develop sooner in more ductile materials as illustrated by comparing the notched 2014 and 2024 aluminium alloy with the 7075 alloy.

The combination of fatigue stresses and even a mildly corrosive environment accelerates the time for crack generation. The effects on later crack growth are even more pronounced. One of the more important environmental constituents is water vapour which has a strong effect on the fatigue of aluminium and its alloys. Previous load history can have two effects. First, if the material has been hardened, as already mentioned in the case with commercial aluminium alloys, the yield stress is increased, and under constant stress conditions, the time to generate a crack would be increased. Secondly, prior loading can produce significant residual stresses at the root of a notch. For example prior tensile stress will leave a compressive residual stress, and cracks at the notch root will be very slow in developing compared to the annealed state. A similar situation may arise by superimposed stress conditions such as mean stresses due to external loads or to residual stresses. Treatments such as shot-peening are used to induce compressive residual stresses on surfaces, so that N_I can be significantly increased.

Before applying any of these guidelines to a specific situation it should first be ascertained whether cracks already may exist, in which case the crack generation state, often referred to as crack initiation stage as well, is effectively bypassed. Many real structures already contain microcracks before the first service load is ever applied.

Material	Specimen Geometry	Crack Site	N_T Cycles	Cracklength at first observation 10^{-2} mm	N_I/N_T
Pure Al	smooth	Grain Boundary	$3 \cdot 10^5$	1.3	0.10
2024-T3	smooth		$5 \cdot 10^4$	10.2	0.40
	smooth		$1 \cdot 10^6$	10.2	0.70
2024-T4	smooth		150	25	0.60
			$1 \cdot 10^3$	25	0.72
			$5 \cdot 10^3$	25	0.88
Pure Al	notched $K_t \approx 2$	slip bands	$2 \cdot 10^6$	0.025	0.005
2024-T4	notched $K_t \approx 2$	inclusions	$1 \cdot 10^5$	2.0	0.05
			$3 \cdot 10^6$	1	0.07
			$2 \cdot 10^3$	6.3	0.015
2014-T6	notched $K_t \approx 2$		$2 \cdot 10^4$	6.3	0.02
			$1 \cdot 10^6$	6.3	0.05
			$2 \cdot 10^5$	50	0.64
7075-T6	notched $K_t \approx 2$	inclusions	$5 \cdot 10^3$	7.6	0.20
7075-T6	notched $K_t \approx 2$		$1 \cdot 10^5$	7.6	0.40

Table 1: Crack generation time as a fraction of total life (after ASTM STP 495, Grosskreuz)

Fatigue Crack Growth

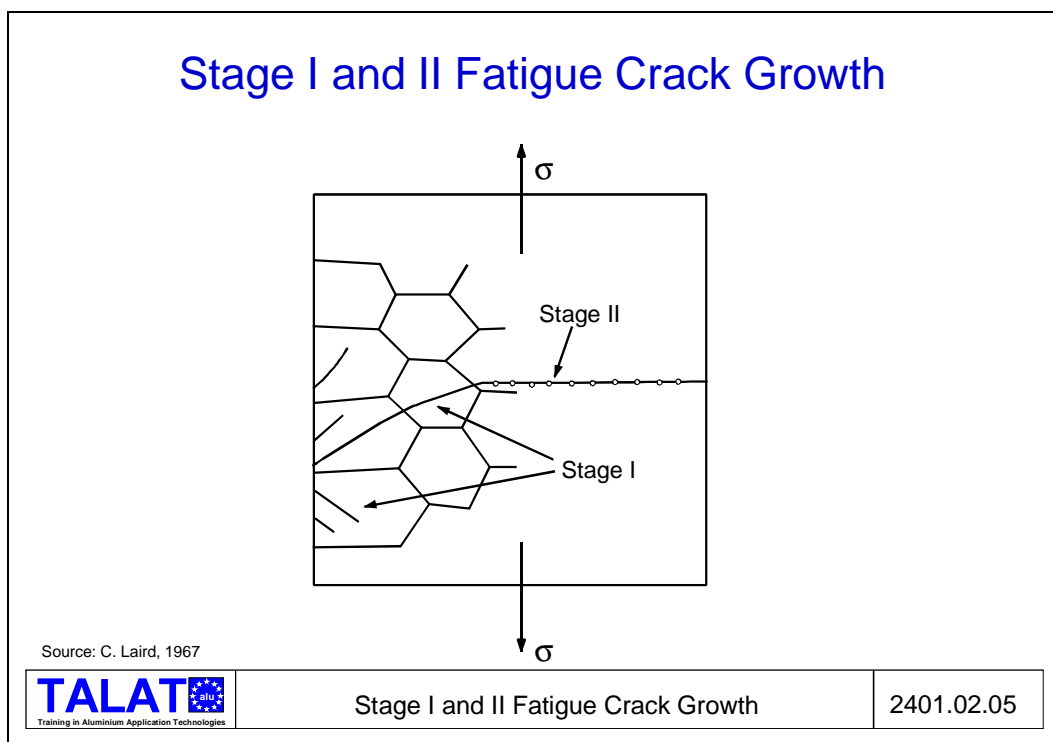
The increase in crack length Δa for the increment ΔN of load cycles define the growth rate, $\Delta a/\Delta N$. This is a function of both, the crack length a and the stress or strain amplitude. Observed growth rates may range between 10^{-10} m/cycle at low amplitudes to about 10^{-3} m/cycle at high amplitudes. The importance of the growth rate lies in its use to calculate remaining life times, given a certain initial crack length a_i after N_I load cycles. Assuming that a is a continuous function of N , the instantaneous crack growth rate da/dN can be used to give the total life for a crack propagation

$$N_P = N_T - N_I = \int_{a_i}^{a_r} \frac{da}{(da/dN)}$$

Desirable is a theory of fatigue crack growth yielding universal expressions for da/dN as a function of a , σ or $\Delta\varepsilon_p$, and material properties. Not only could such a theory be used to predict fatigue lives, but it would allow a designer to choose those materials most resistant to fatigue crack growth. Central to the construction of such a theory is a model which can be described mathematically to give the incremental advance of the crack. Such a model must be derived from experimental observation.

Respective analytical expressions, supported by experimental observations, in the case of aluminium alloy structural components and their welded connections, as well as the analytical procedures for life estimation will be explained in detail under a later chapter on fracture mechanics applications. At this point some general observations on the propagation modes, the crack growth mechanisms and the effects of multiple load amplitudes and other parameters on da/dN will be briefly covered.

Cracks forming in slip bands propagate along the active slip planes which are inclined at $\pm 45^\circ$ with respect to the tensile stress axis. This shear mode propagation, Stage I growth, tends to continue more deeply into the specimen the lower the amplitude of loading. The crack soon begins to turn and follow a course perpendicular to the tensile axis. This tensile mode propagation as called Stage II growth, characterising crack growth up to the critical length for which the next load peak produces tensile failure of the specimen (**Figure 2401.02.05**).

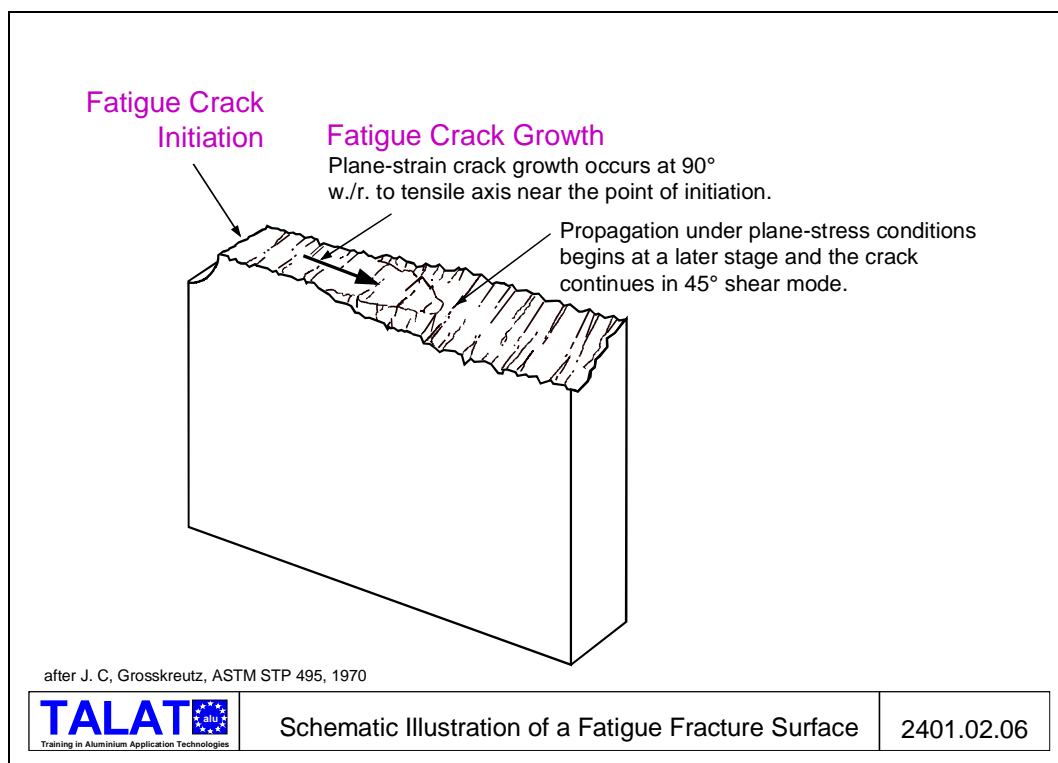


Actually, the Stage I growth in a polycrystalline material involves hundreds of individual slip band cracks linking up to form a dominant crack at about the time Stage II growth begins. Stage II crack growth life increases with increasing load amplitude.

Cracks formed at inclusions grow only a few micrometers in Stage I before changing to the Stage II mode. At low stress or strain amplitudes, very few inclusion cracks are generated and one such crack may grow all the way into the final failure crack. At higher

amplitudes several inclusion cracks may joint together to form the final crack, thus producing discontinuous jumps in da/dN . This case is of importance and will be demonstrated in more detail in the respective chapter on fracture mechanics applications to welded connections.

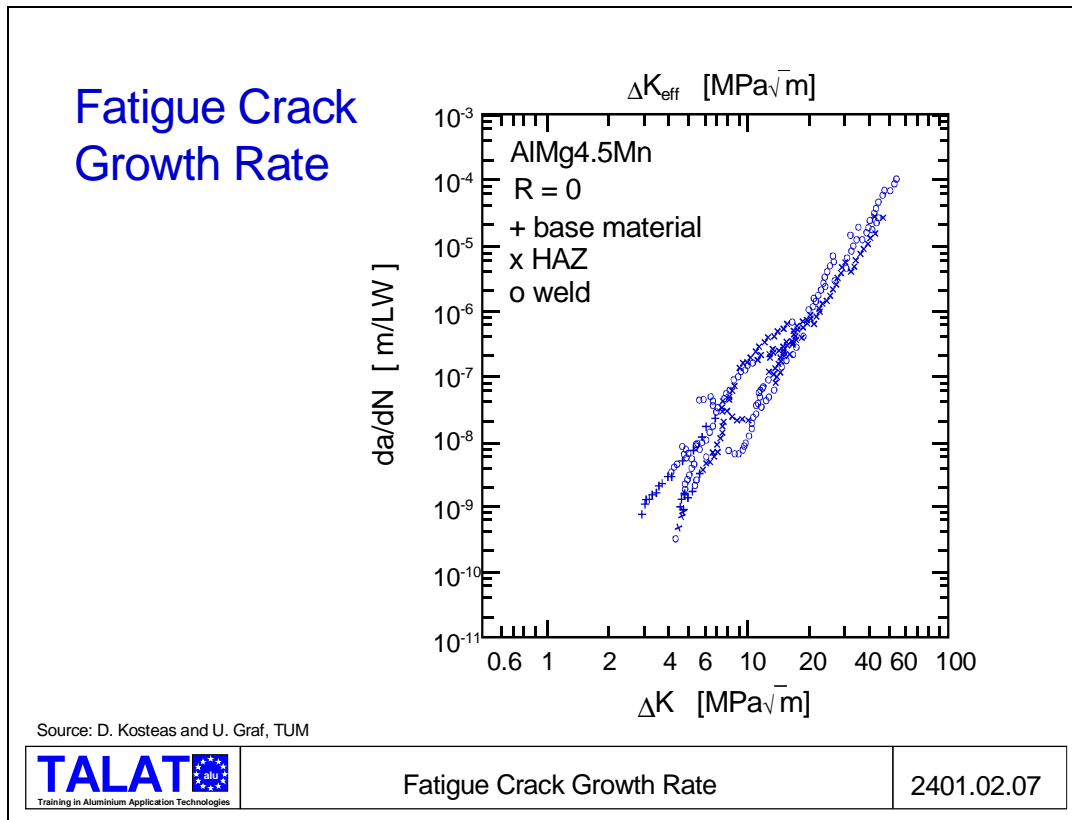
Examination of fracture surfaces tells us a lot about the mechanism by which the crack advances. For all practical purposes the entire fracture surface formed is governed by the Stage II mode. Very near the crack nucleus, while the crack length is still small, conditions of plane strain hold at the crack tip. The fracture surface is microscopically flat and oriented perpendicular to the tensile axis. As the crack grows in length a shear lip begins to develop where the fracture surface intersects the specimen surface (**Figure 2401.02.06**). This reorientation of the fracture surface to a 45° position with respect to the tensile axis is caused by the plane stress conditions at the crack tip intersecting the surface. As the plastic zone in front of the crack tip increases in dimension to become comparable to that of the specimen thickness, plane stress conditions hold everywhere at the crack tip and the fracture surface is one continuous shear lip or double shear lip.



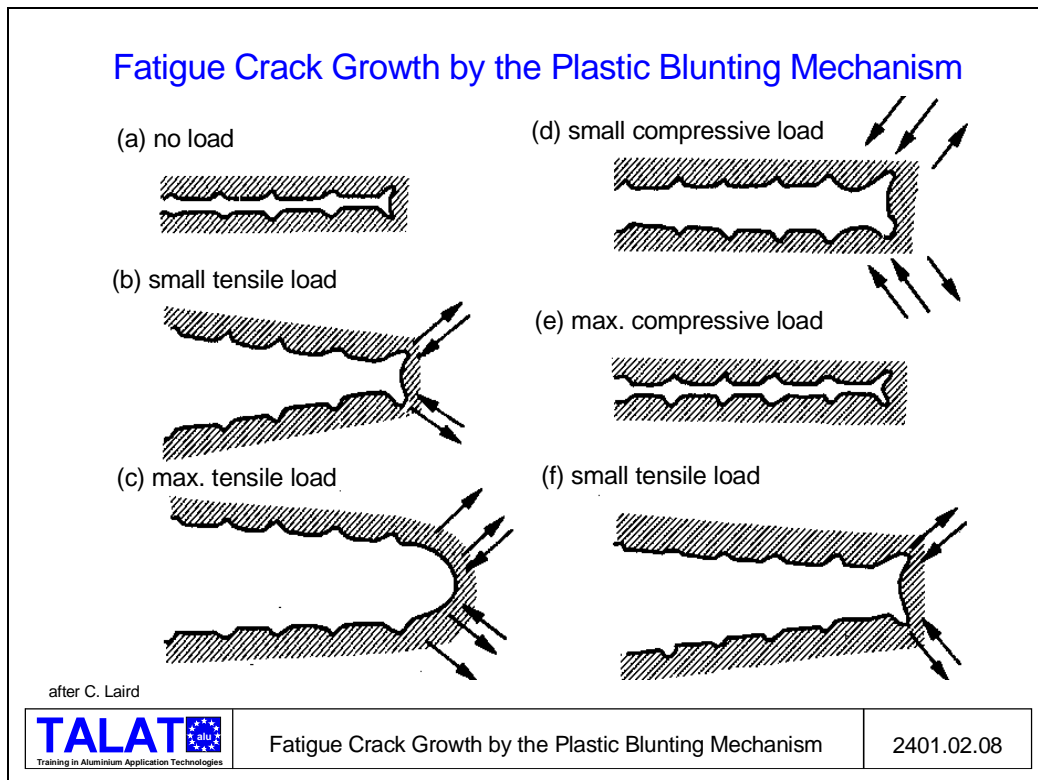
The flat, plane strain surface is especially rich in detail, exhibiting regularly spaced striations, even visible at optical magnification. Each striation represents the crack advance for one cycle of load and this was verified experimentally. The conclusion that each striation corresponds to one cycle of crack advance allows one to measure the local rate of crack growth by measuring the distance between adjacent striations. Careful experiments have shown that these striations spacings correspond very closely to the microscopic rates of crack growth measured on the surface of the specimen. However the first is consistently larger than the microscopic da/dN on the surface which indicates that surface growth is only an average of local rates in the general direction of Stage II growth.

Crack Growth Mechanisms

Crack growth rates can be expressed analytically for different crack lengths and for different loads in terms of the stress intensity factor range ΔK as will be shown later on. The correlation of crack growth rates with a crack tip stress intensity factor provides the key to a study of crack growth mechanisms. It is only recently that such correlations have been experimentally verified on a reliable basis for aluminium alloys and especially the material zones in aluminium weldments (**Figure 2401.02.07**).



Cross sections taken through the crack tip at various parts of a load cycle have established that Stage II growth occurs by repetitive blunting and re sharpening of the crack tip (**Figure 2401.02.08**).

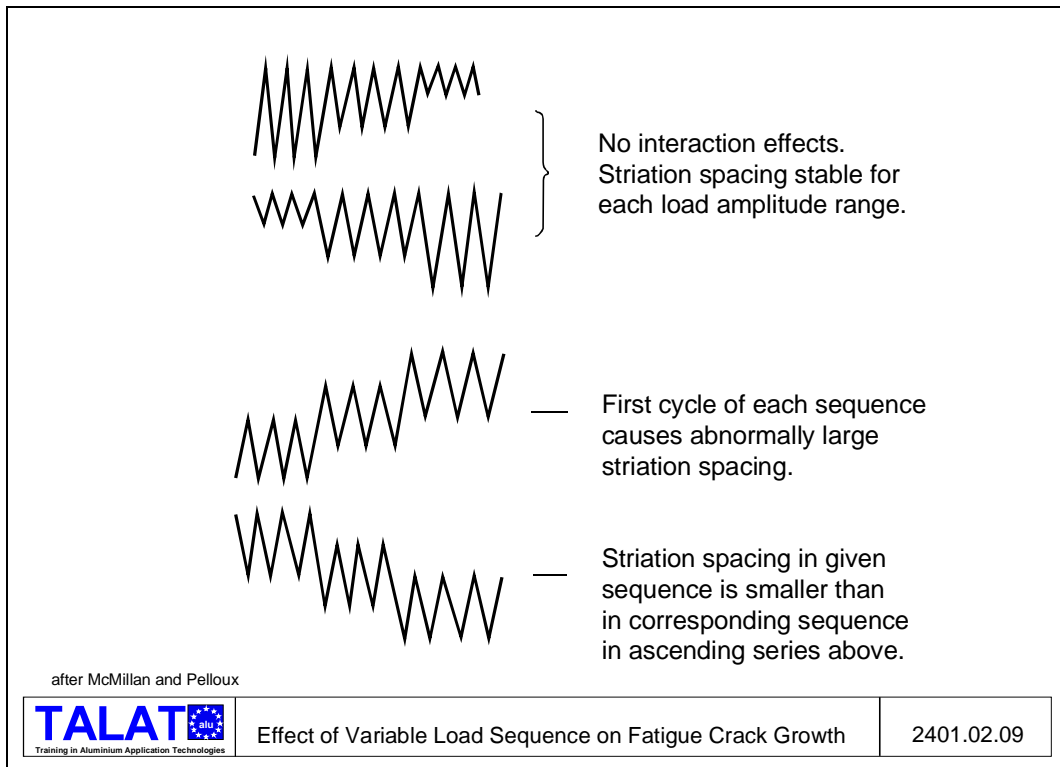


Although many investigations of the micromechanisms of fatigue crack growth under spectrum loading have been undertaken in the last decades as there is still an urgent need for further work. However, results are available allowing to some general statements on the effect of multiple load amplitudes.

In the first place it is important to distinguish between the effects of spectrum loading on total life on a laboratory specimen and on the growth rate of an existing crack. Much of the literature of spectrum loading has dealt only with total life and the concept of cumulative damage. Because total life includes both crack initiation and crack propagation, we cannot expect to apply such results to understanding the effect of multiple loads on just crack growth.

The effects of multiple load amplitude can be best understood in terms of crack tip plastic deformation. The concept of stress intensity factor is of limited use here because it relates only to elastic stresses. The two important concepts are localised work hardening and localised residual stresses at the crack tip. More on the practical application of such analytical and experimental results will follow under the chapter on fracture mechanics, especially in defining a so-called effective stress intensity range.

A particularly instructive set of experiments demonstrated in **Figure 2401.02.09** depicts the effect of various load sequences on the microscopic growth rate, as determined in experiments on 2024-T3 aluminium, from striation spacings on the fracture surface. Keeping the maximum load constant and varying the amplitude range had no significant effect on crack growth. Reversing the sequence did not affect the results either. If the load amplitude range was kept constant and the maximum load was varied, interaction effects were observed.



The importance of separating crack initiation and crack propagation interaction effects was stressed. Intermittent overloads can delay the subsequent growth of cracks at lower loads. On the other hand, such overloads are capable of generating new cracks, which can then grow on the lower amplitudes. The overall effect would be to shorten the effective service life of the member by linking together many individual cracks generated in this manner. An opposite case is the so called 'coaxing', in which fatigue at low amplitudes followed by high amplitudes leads to longer, overall lives, even though such a sequence can produce larger than normal crack growth rates. In this case nucleation of cracks is suppressed by the coaxing procedure which hardens the surface layers.

Effect of other Parameters on Crack Propagation Rate

Crack growth rates are affected by temperature, environment, strain rate or frequency, and material properties.

Raising the temperature usually promotes dislocation and slip processes so that the plastic blunting mechanism can act more freely, thereby increasing the rate of Stage II crack growth. Because Stage II crack mechanisms are governed largely by unidirectional mechanical properties (modulus of elasticity, yield and ultimate strength, strain hardening coefficient), the effect of temperature of these properties can be extrapolated directly to crack growth rates. However, structural engineering components rarely have to operate at such elevated temperatures.

Environment has a significant influence on crack growth rate and may affect the mechanism of fracture. The presence of a corrosive environment often will change a ductile fracture mode into a brittle one. The effect is for environmental attack to increase the

rate of crack growth. It is not uncommon for the crack growth rate to increase by a factor of about 10 in aluminium alloys exposed in humid air compared to the rate in vacuum. Attention should be given however to the interpretation of laboratory tests on the environmental effects on fatigue and their extrapolation to service conditions. The frequency of loading and the severity of the environmental attack cannot be estimated reliably without prior experimental verification.

The effect of material properties on da/dN is of great importance because such information can be used by the engineer to choose crack resistant materials. Such information is unfortunately not documented in an easily and retrievable and adequate manner. The research in this field is still going on. It can be stated that Stage I crack growth is more rapid in wavy slip mode materials like aluminium. The growth rate in Stage II is governed as already mentioned by the unidirectional properties of the material. Slower rates are obtained by raising the modulus of elasticity, the ultimate tensile strength, and the strain hardening rate. The effect of small grain size in reducing crack growth rates is considerably at low stress amplitudes but negligible at large amplitudes. It should be mentioned that significant variations in fatigue crack propagation rates may be observed in the same material obtained from different sources.

Endurance Limit

The endurance limit or fatigue limit, is recognised as a change in the slope of a conventional S-N curve from negative to zero (flat). Aluminium and its alloys does not give S-N curves with strictly zero slopes at long lives. Nevertheless the curve is nearly flat, and it is usual to speak of the endurance limit at 10^7 or 10^8 cycles. The definite endurance limit is connected strongly with the existence of a sharp yield point in the tensile stress-strain curve, as for instance in the case of iron or steel. On the other hand in the case of aluminium alloys, dislocation locking does not occur - which is associated with the sharp yield point - so that there is no definite stress below which cracks refuse to grow.

Predictive Theories of Fatigue

When will a given construction or component fail?

The design engineer needs the answer to provide safe design. The operator demands the answer to provide safe operation. The question can usually be answered through a combination of past experience, experimental inspection and testing, and semi-empirical fatigue theories. While the latter provide only approximate answers, they are especially important to the design engineer.

Damage Accumulation Theories

The first theories of fatigue were aimed at describing the typical constant amplitude S-N curve. In terms of the accumulation of some vague, undefined damage within the material. Failure was assumed to occur with the accumulation of a critical amount of damage. The parameters which had to be adjusted were the rate of damage accumulation and the critical amount for failure. A useful extension of this concept was made by Palmgren-Miner to the case of variable amplitude loading. The criterion for failure under a series of different load amplitudes is given as

$$\sum_i \frac{n_i}{N_i} = 1$$

the linear damage accumulation rule.

Detailed information on the Palmgren-Miner rule and its applicability in practice will be given in **Lecture 2401.04** dealing with load and stress spectra and damage accumulation. Comparison of the rule with experiment has shown it to err in most cases, often on the unsafe side. Yet the rule is close enough to reality, so it is being used frequently as a guide when other more precise information is lacking. That such a simple concept of damage accumulation works at all lies in the basic nature of fatigue failure. Whether the basic process is crack generation or crack growth, each proceeds by a definite increment in each cycle of stress. Crack generation proceeds until a critical stage is reached such that crack growth can then proceed. The crack propagates until it reaches a critical length, such that the next load application brings catastrophic failure. In each case, the term damage can be substituted for crack generation or growth, and the critical stage of the development can be termed a critical amount of damage accumulation. Assuming no interaction effects between different load amplitudes leads directly to the above linear damage accumulation rule.

Manson-Coffin Law

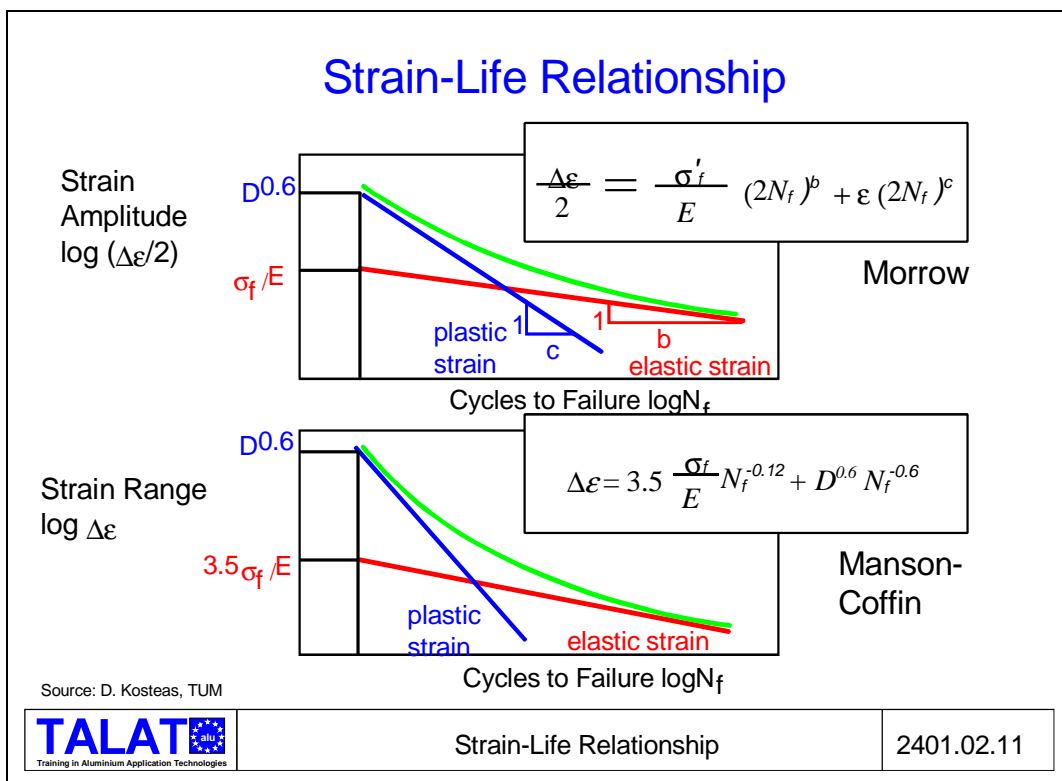
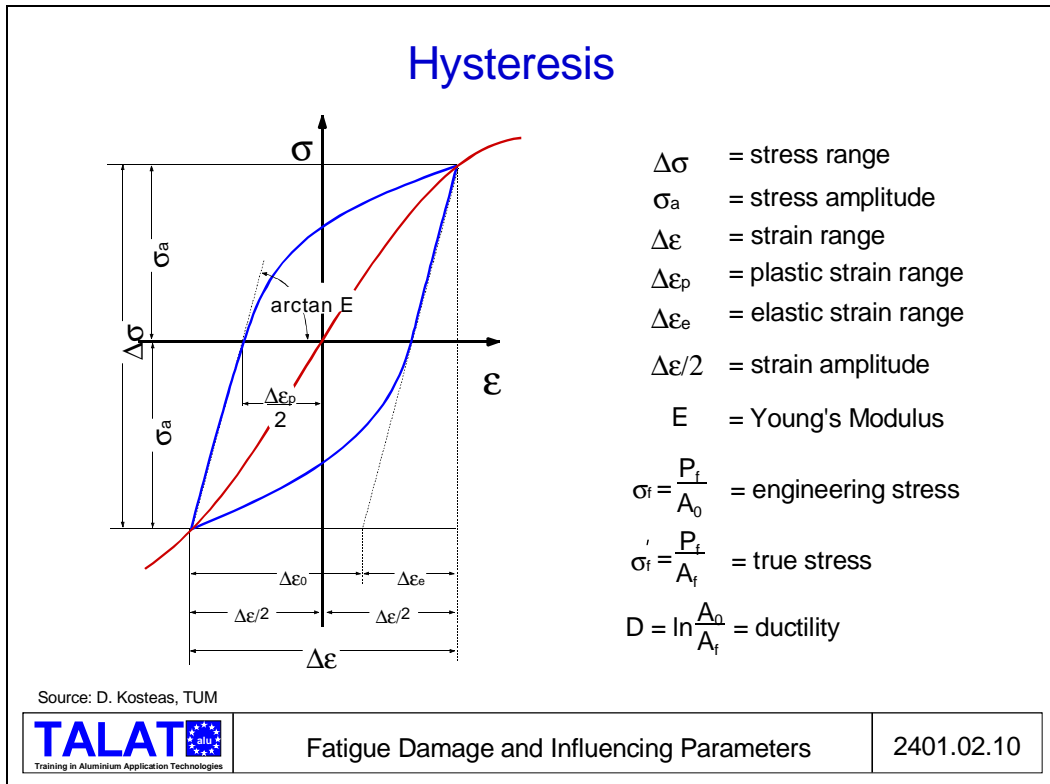
Under conditions of constant plastic strain range $\Delta\epsilon_p$ the numbers of cycles to failure is found to be (for $\Delta\epsilon_p > 0.01$)

$$\Delta\epsilon_p \cdot (N_T)^z = C$$

where C and the exponent z are material constants, z weakly dependent on material and with a value in the order 0.5 to 0.7 and the constant C related to ductility and the true tensile fracture strain. This general relationship has been modified to include the total strain range and is usually referred to as the Manson-Coffin law.

The development of numerical analysis procedures, especially the finite element method allow a rapid and accurate analysis of local stress and strains at notches or cracks. Related to the true strain behaviour of the material under repeated loading - the cyclic stress-strain curve as compared to the initial monotonic, static stress-strain curve - the possibility of applying such relationships especially in the area of low-cycle fatigue are outlined in the following. Some more details on local stress concepts in fatigue will follow in

Lecture 2401.05. The overall procedure of analysis and life estimation through the Manson-Coffin relation is outlined in **Figure 2401.02.10** and **Figure 2401.02.11**.



When plotted on log-log paper, the relationship becomes a straight line. With the aid of the cyclic stress-strain relation this plot can be converted to a S-N curve. A useful expression for the exponent z is given in terms of the cyclic hardening exponent n'

$$z = \frac{1}{2n' + 1}$$

Since $n' \approx 0.15$ we get $z \approx 0.75$, this value being about 25% larger than measured values. The detailed dependence of C and z on material constants is not resolved completely. In our opinion the Manson-Coffin relation and further derivations may be a useful tool for comparative studies and initiation life estimations for different materials. The advancement of life estimation methods on the basis of fracture mechanics, especially in cases where fatigue life is governed mainly by propagation, is nowadays a more powerful and sophisticated tool in the hands of a knowledgeable design engineer.

Crack Growth Laws

Many empirical crack growth laws have been proposed. The most popular have the form

$$da/dN = C \cdot F(a,s)$$

where C is a constant which depends on the material and $F(a,s)$ is a function determined empirically from the data. There are two important ingredients which are necessary to any successful crack growth theory. First, a realistic model for crack growth. Second, analytical methods for expressing the model in mathematical terms so that a quantitative relation may be derived. It is worth noting that theoretical crack growth laws are of the form

$$da/dN = C \cdot K^m$$

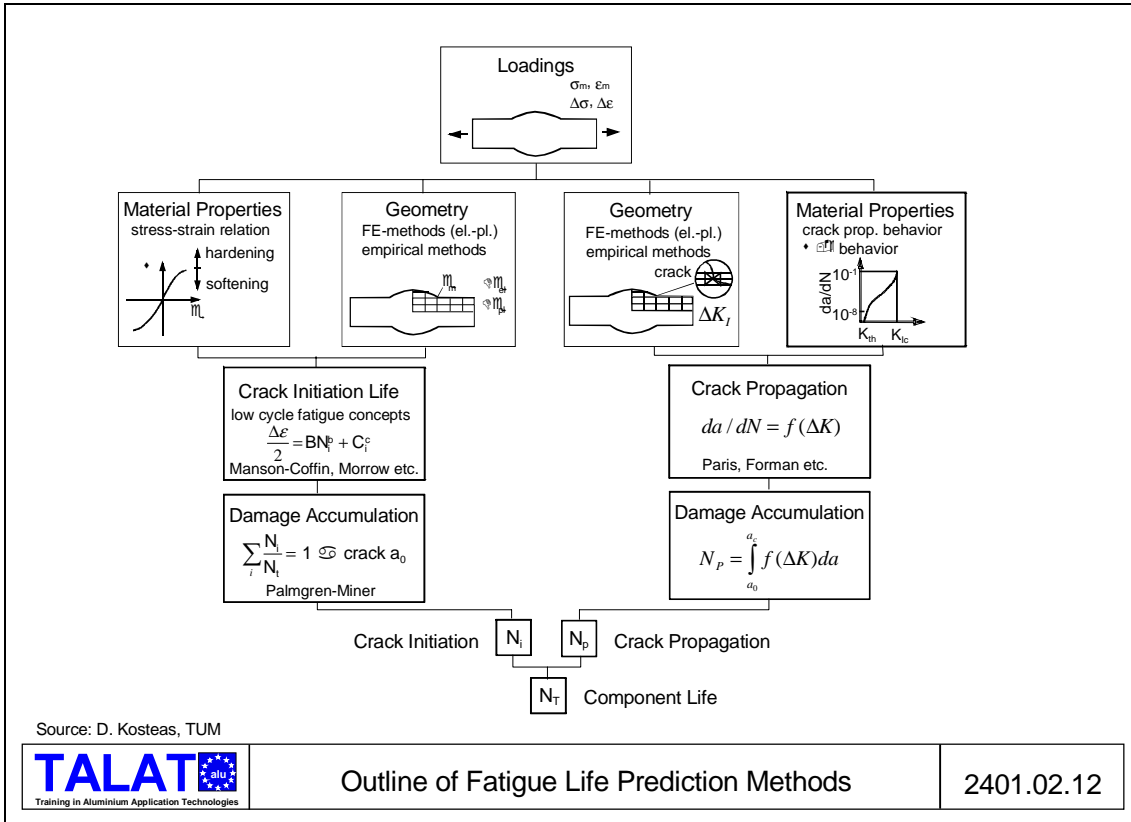
where C is again a material constant and K is the stress intensity factor depending on the instantaneous crack length and the overall nominal stress and affected by geometrical and loading parameters. This is the form found empirically by Paris and others and is discussed in more detail under Chapter 12 on fracture mechanics and life estimation.

Ideal Cumulative Damage Theory

Ideally one would like to predict the failure of structural components which are subject to a spectrum of loads in a variable environment. While such a theory is not fully available the framework for achieving it is fairly clear. Fatigue mechanisms provide the basic idea, namely that the number of cycles to failure is the sum of the crack initiation time N_I and the crack propagation time N_P (see **Figure 2401.02.12**)

$$N_T = N_I + N_P$$

Quantitative expression of N_I is a difficult problem, the effect of aggressive environment on crack initiation time is also largely unknown. However, if $N_I \ll N_P$ then the problem is minimised. This is often the case with structural components containing substantial notches, for instance welded structures.



Such an approach to cumulative damage and the prediction of fatigue lives requires an accurate model for the simulation of the service life of the structure as well. Such estimations are performed almost entirely nowadays by computer simulation.

2401.03 Fatigue Data Analysis and Evaluation

- Analysis of data
- Analysis in the middle-cycle fatigue range
- Analysis in the high-cycle fatigue range
- Fatigue diagrams
- Linear P-S-N curves
- Non-linear P-S-N curves
- Some problems of data analysis in practice

It is a requirement for fatigue design stresses to be related to some probability of failure. This is particularly true in the context of structures whose failure could be catastrophic but it also arises in relation to design rules which are based on limit states. A need for statistical analysis also arises during an actual design procedure, when further levels of sophistication beyond the rules of a recommendation have to be established. In such cases own data or comparisons of data have to be performed on a homogeneous way and compatible to the procedures of the recommendations themselves. In order to meet such requirements, fatigue data need to be analysed statistically.

Fatigue data have long been the subject of statistical considerations because of their inherent scatter. However, in practice, fatigue data, especially for welded joints, are seldom ideal material for statistical treatment. Either in the case of literature data, which usually come from several sources, and need to be analysed to provide the basis for design rules - or in the case of data from special fatigue tests carried out under well defined and controlled conditions to validate a particular design. Despite the merits of computerised analysis it must be reminded that the ability to make a qualified, well weighted decision, based on engineering judgement (whatever this may be more than experience) is still extremely valuable. We normally encounter three different types of data:

- many results for identical specimens made and tested at the same laboratory
- many results for similar specimens, representing more severe stress concentrations than in the first type, obtained mainly at one laboratory over a period of years
- many results for similar specimens obtained from many different investigations over a period of years.

No simple statistical method is suitable for treating all three types of data. Further more, the greatest problem is presented by the third type; this is the type of data which need to be evaluated in the formulation of design rules.

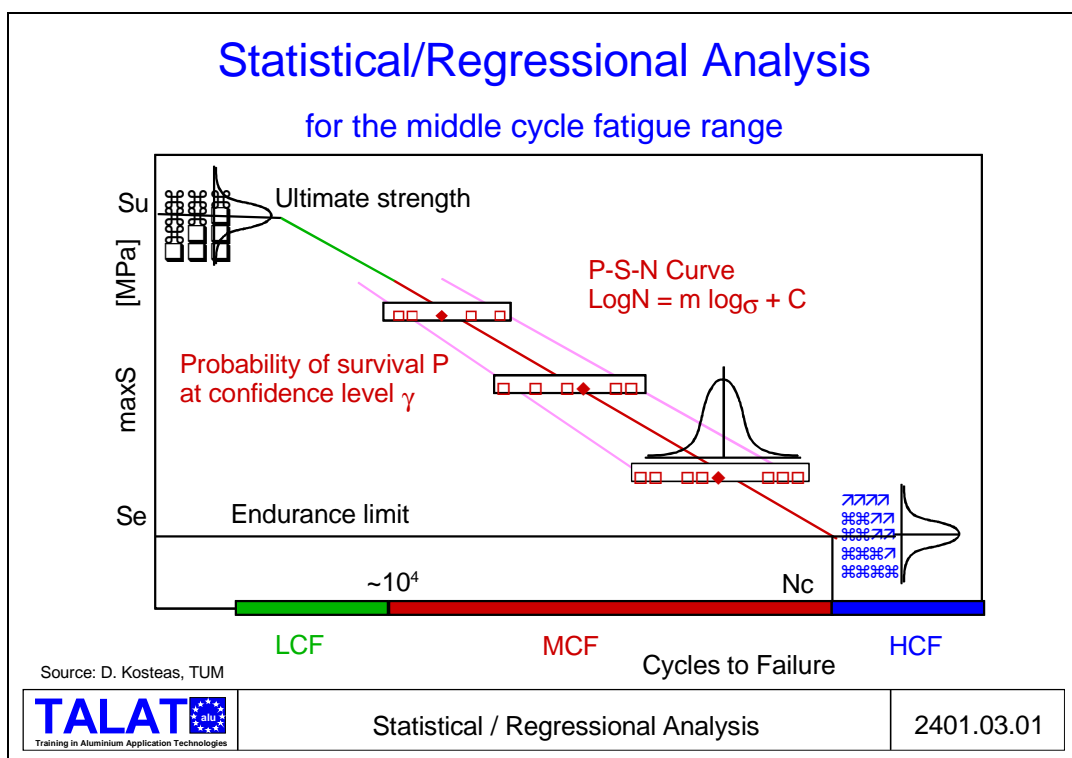
Analysis of Data

Fatigue life of a specimen will be expressed in cycles to failure. This is the observed variable dependent on the investigated property, i.e. fatigue strength. Practically we perform a test data analysis in order to calculate characteristic values for a statistical population out of the random sample observation. These are the *mean value* \bar{x} and the *variance* s^2 or the *standard deviation* s which identify the population and enable further comparisons.

As a next element we need *confidence intervals* for the observed data which again describe the relation of the sample to the whole population. In the case of fatigue test data we are interested in the calculation of probability of failure or *probability of survival* limits.

Through comparative evaluation of test results by means of *variance analysis* significant or non-significant deviations may be calculated. Finally the functional relationship between two variables, here fatigue strength and cycles to failure, has to be established by using methods of *regression analysis*.

It is pointed out that statistical and regression methods are included in the operations of the 'Aluminium Data Bank' installed at the Department Aluminium Structures and Fatigue of the University of Munich. Details on these procedures are given in **Lecture 2403.02**.

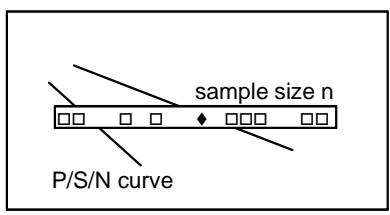


Analysis in the Middle-Cycle Fatigue Range

For most practical applications a fatigue life range approximately between $1 \cdot 10^4$ and $5 \cdot 10^6$ cycles may be defined as a range where the relationship between cycles to failure and fatigue strength will be linear for all practical purposes, when the data are plotted on a double-logarithmic scale, see **Figure 2401.03.01**. **Figure 2401.03.02** describes a calculation of important statistical parameters. In **Figure 2401.03.03** an outline of statistical distributions is given pertaining to the calculation of different parameters or to the performance of statistical evaluations and comparisons. Further details can be found in respective literature.

Fatigue Test Data Analysis

Statistical Analysis



$\log \sigma$

$\log N$

sample size n

P/S/N curve

N is the dependent variable
 σ is the independent variable

Regression analysis
estimation of regression parameters through minimisation of deviations (least squares)

linear S/N: (Wöhler) $\log N = a \sigma + b$ (Basquin) $\log N = a \log \sigma + b$

or non-linear S/N: (Stüssi) $\sigma = \frac{\sigma_z + bN^a \sigma_D}{1 + bN^a \sigma_D}$ (Weibull) $N = B \left(\frac{\sigma_z - \sigma_d}{\sigma - \sigma_d} \right)^{\frac{1}{a}} - B$

Source: D. Kosteas, TUM

mean
standard
deviation

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$


conf { $\bar{x} - a \leq \mu \leq \bar{x} + a$ }

confidence
intervals for
mean and
standard
deviation

$$a = s \frac{c(t; \gamma)}{\sqrt{n}}$$

conf { $a_2 \geq s \geq a_1$ }

$$a_1/a_2 = \sqrt{(n-1) \frac{s^2}{c_{1/2}(\chi^2; \gamma)}}$$



Fatigue Test Data Analysis

2401.03.02

The observed event or the element of the test sample, i.e. the dependent variable, is the logarithm of the number of cycles to failure $\log N$. Fatigue strength is regarded as the independent or controlled variable S or $\log S$. Although not used frequently in structural engineering anymore the possibility of expressing fatigue behaviour in a linear-logarithmic relationship S vs. $\log N$ is still mentioned. This method has nowadays been abandoned in favour of the double logarithmic relationship $\log S$ vs. $\log N$. For every sample value $x_1 (= \log N_1) \leq \dots x_i (= \log N_i) \leq \dots x_n (= \log N_n)$ we get a respective value for the

$$\text{probability of survival } p_{s,i} = 1 - \frac{i}{n+1} \quad \text{and} \quad (1)$$

$$\text{probability of failure } p_{f,i} = \frac{i}{n+1} \quad (2)$$

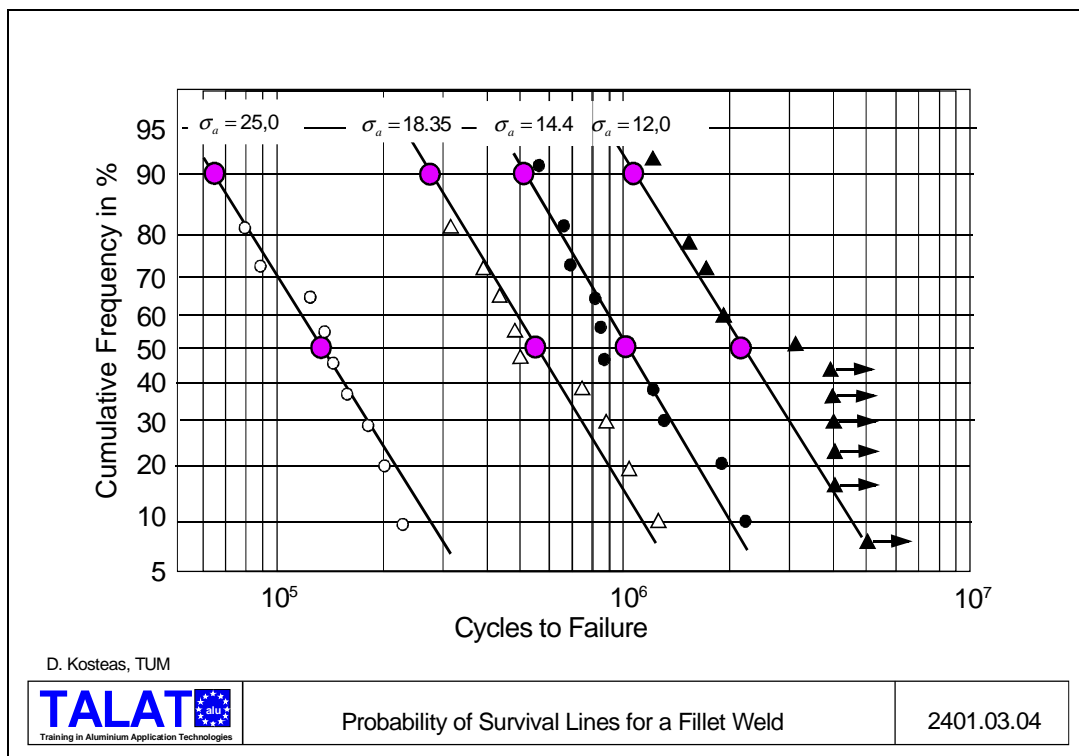
$$\text{with } p_{s,i} + p_{f,i} = 1$$

Distribution	Hypothesis: F(x) is the distribution of the whole population ω^2 -Distribution in Kolmogoroff-Smirnow-Test probability paper						
Confidence Interval	Distribution Parameters			Regression Analysis		Correlation	
	Mean Value Variance σ^2		Variance	Regression Coefficient (slope)	Mean Value	Linearity	Correlation Coefficient
	known	unknown	ω^2 -Distrib.	t-Distribution	t-Distribution		Normal Distribution
Normal Distribution	t-Distribution						
Tests	Comparing Two Samples			Hypothesis $\beta = \beta_0$			Hypothesis $\theta = 0$
	Hypothesis: $\mu = \text{mean}$		Hypothesis $\sigma_1 = \sigma_2$				
	Test of Hypothesis by Means of Analysis of Variance						
	Comparison of Several Samples			Hypothesis $\beta = 0$			F-Distrib.
Hypothesis: $\mu_1 = \dots = \mu_n$		F-Distrib.					

D. Kosteas, TUM $\mu = \text{mean}$ and $\beta = \text{slope of population}$

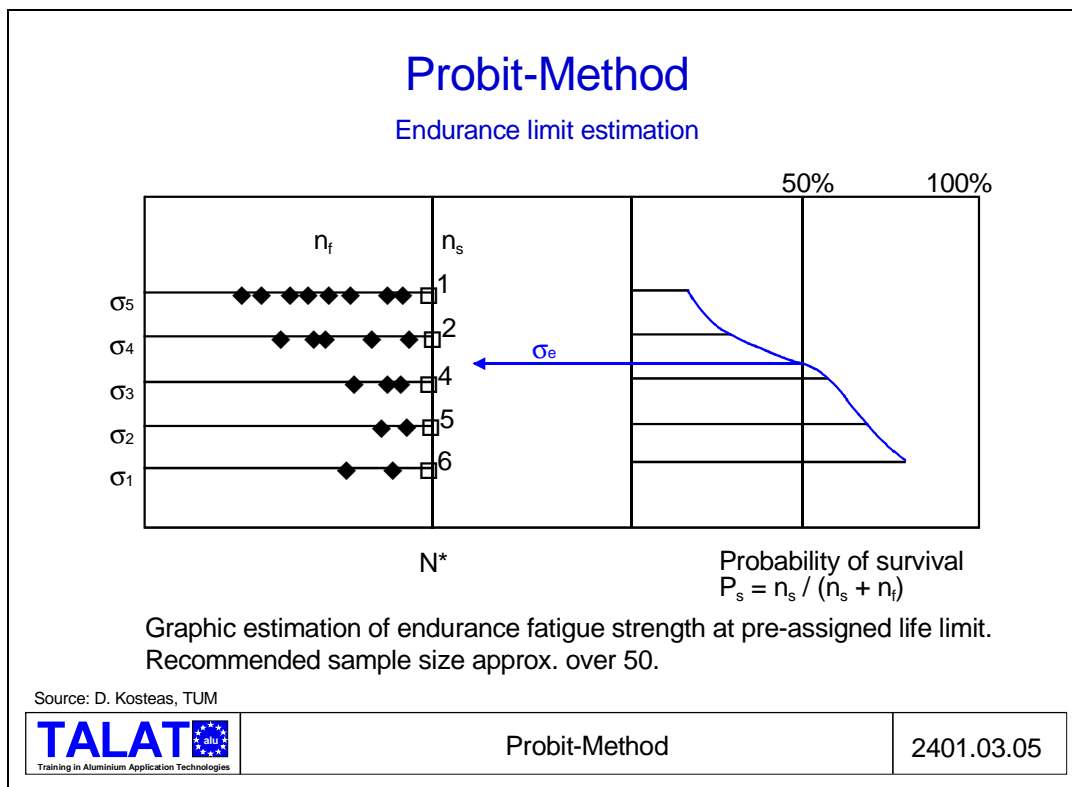
TALAT Training in Aluminium Application Technologies Statistical Methods for Calculations of Distributions and Significance 2401.03.03

A normal distribution of the logarithms of the cycles to failure is assumed for the sample and this would result in a straight line for the relationship between the cumulative frequency and the number of cycles to failure if the respective values are recorded on probability paper with a logarithmic scale for the observed number of cycles, e.g. **Figure 2401.03.04**.



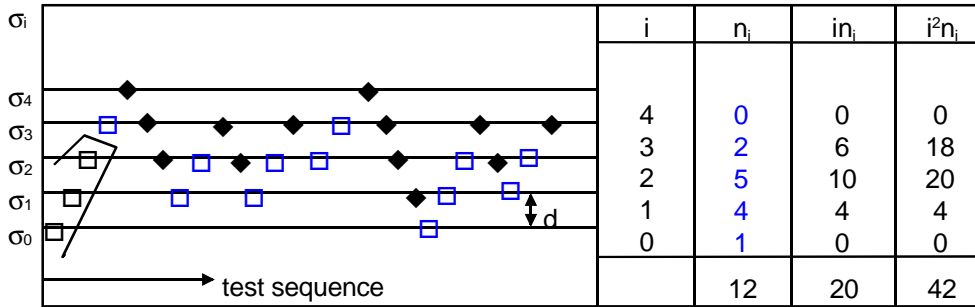
Analysis in the High-Cycle Fatigue Range

The existence of a fatigue endurance limit is discussed in **Lecture 2401.02**. A physically oriented explanation is also given in **Lecture 2403** on fracture mechanics. Here we are involved with the fact of a fatigue life limit, where the slope of the relationship $\log S$ vs. $\log N$ becomes increasingly shallower and failure occurs after increasingly larger number of cycles. Out of practical reasons a limit in cycles to be observed will be set for tests conducted on different fatigue strength levels and the quantity observed will be failure or non-failure, i.e. run-out. This results practically in a distribution of fatigue strength values at the selected cycle limit. Such experimental tests are by nature very costly, especially because of the high costs of testing and the long testing periods necessary. A larger sample size is also needed, which in the case of component or full size testing cannot be supplied. In terms of a testing methodology we distinguish between two test methods, differing in both the process and evaluation: the probit-method (see **Figure 2401.03.05**) and the staircase-method (see **Figure 2401.03.06**). Some other related test methods have been developed, eg. the „arcsin \sqrt{P} “-method or the „Two-Point“ method, see **Figure 2401.03.07**. In practice the staircase method gives reliable results for the mean of the fatigue strength with a total of 25 specimens and can be recommended.



Staircase-Method

Endurance limit estimation



Pre-assign limit cycle life N^* ; "select" step size d ; sample size approx. 25

Exclude tests up to first pair of contrary results (no-failure \square ; failure \blacklozenge)

Sum up "no-failures" and "failures" and continue evaluation with results of event with smaller sum (in this example this is the event of "no-failures": 12 points against the event "failures": 13 points)

Estimate the mean of fatigue endurance at N^* cycles:
$$\sigma_e = \sigma_0 + d \left(\frac{\sum in_i}{\sum n_i} + \frac{1}{2} \right)$$

(use "-" for "failure", "+" for "no failure")

Estimate the standard deviation ('unreliable'):
$$s = 1.62d \left(\frac{\sum n_i \sum i^2 n_i - (\sum in_i)^2}{\sum n_i^2} + 0.029 \right)$$

Source: D. Kosteas, TUM

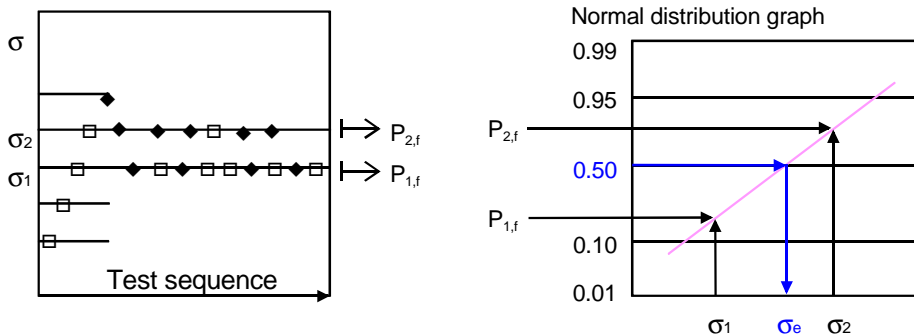


Staircase-Method

2401.03.06

Two-Point-Method (Little)

Endurance limit estimation



Testing and evaluation for a given limit fatigue life N^* :

Begin testing as in staircase-method until two stress levels with a probability of failure P other than 0 or 1 are established.

Concentrate further test specimens on these two levels.

Estimate mean for fatigue strength at the given life limit N^* graphically.

Source: D. Kosteas, TUM



Two-Point-Method (Little)

2401.03.07

Fatigue Diagrams

Results of fatigue tests can be depicted in various diagrams depending on the choice of parameters. **Figure 2401.03.08** shows some common diagrams used in structural engineering. The Smith- and the Haigh diagrams are traditionally more frequently used in

mechanical engineering and they involve the stress amplitude plotted over mean stress. In civil engineering it is traditional to use the relationship stress vs. cycles to failure. Its earlier form - a linear-logarithmic relationship between maximum stress and cycles to failure, also called the Wöhler curve - has been substituted by a double logarithmic relationship of the stress range $\log \Delta \sigma$ vs. cycles to failure $\log N$ in the last decades, especially for welded structures.

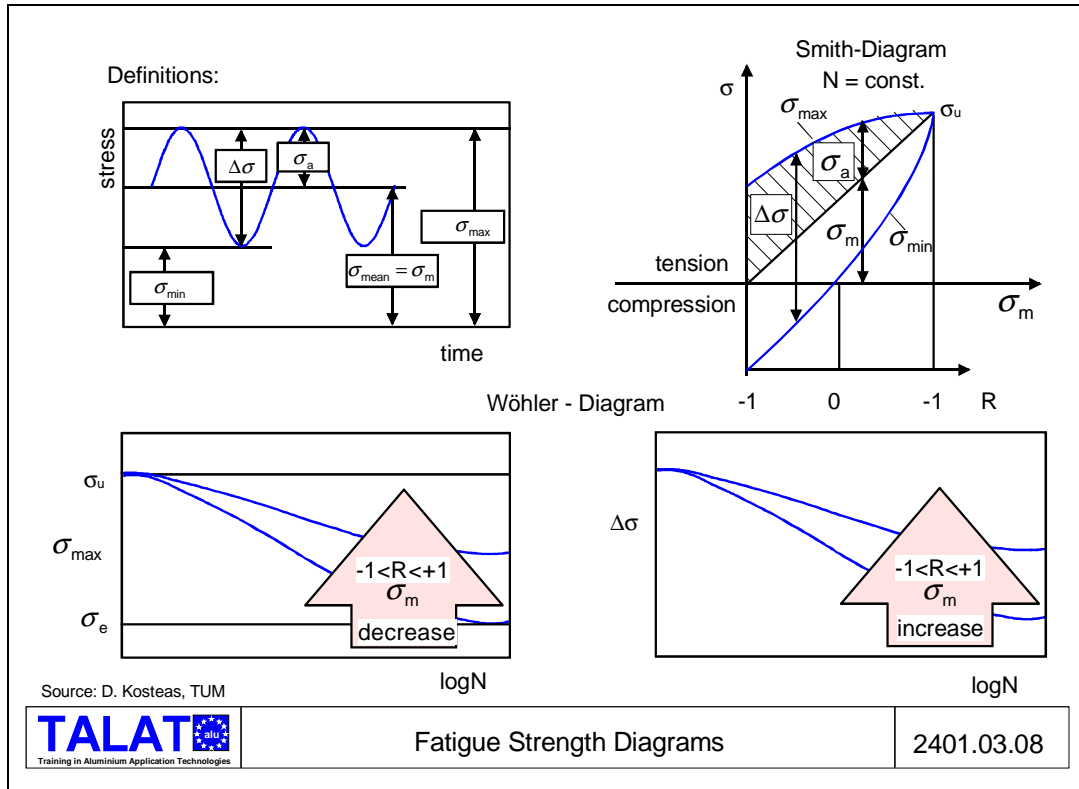
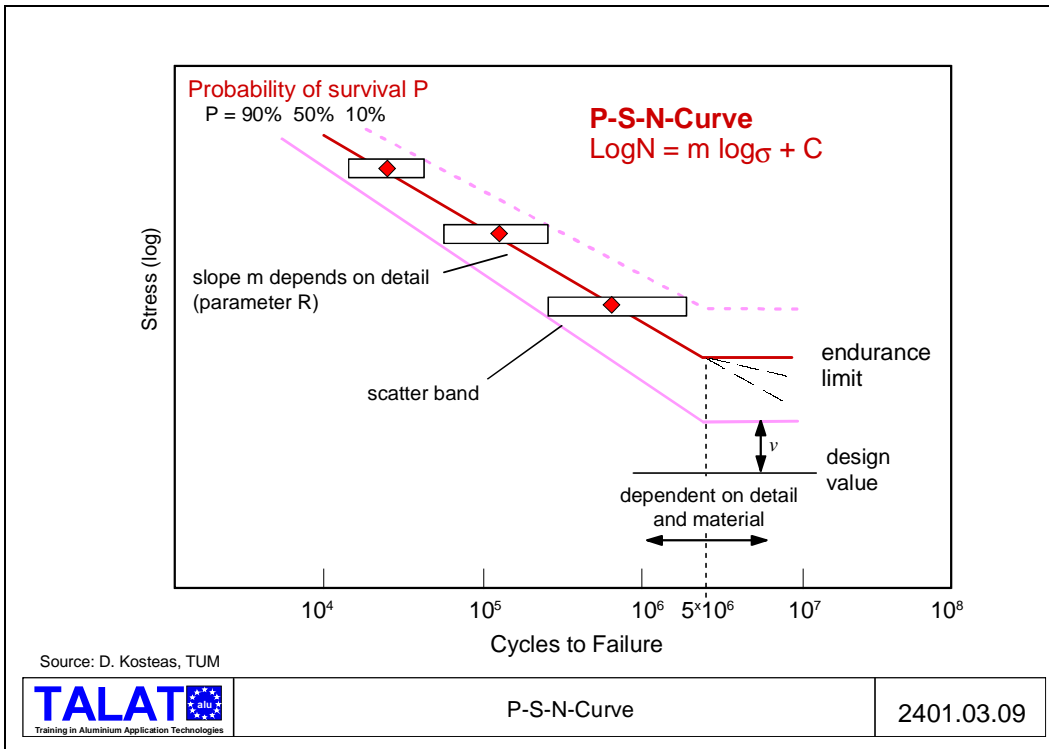


Figure 2401.03.09 shows the general form of this relationship which can be calculated by regression analysis and the Gauß method of least squares. Straight line probability of survival limits, either in the traditional form as in **Figure 2401.03.01** with increasing scatter band width for increasing number of cycles, or in the now usual form of limit lines parallel to the mean line will be similarly calculated from respective probability of survival values. These curves are often referred to as P-S-N (Probability - Stress - Life) curves.



Linear P-S-N Curves

The double logarithmic relationship (sometimes referred to as the Basquin curve) is given in the following form

$$N = C \cdot \sigma^{-m} \quad \text{or} \quad (3)$$

$$\log N = -m \cdot \log \sigma + \log C \quad (4)$$

Given k pairs of values for the variables $y = \log \sigma$ and $x = \log N$ for a specific probability of survival value we get the following equations for the

$$\text{slope} \quad m = \frac{k \cdot \sum_{i=1}^k (x_i \cdot y_i) - \sum_{i=1}^k (x_i) \cdot \sum_{i=1}^k (y_i)}{k \cdot \sum_{i=1}^k (y_i^2) - \left[\sum_{i=1}^k (y_i) \right]^2} \quad (5)$$

$$\text{intercept} \quad \log C = \frac{\sum_{i=1}^k (x_i) - m \cdot \sum_{i=1}^k (y_i)}{k} \quad (6)$$

Non-Linear P-S-N Curves

The previously defined linear double logarithmic relationship may be used successfully for the representation of test results in the middle cycle fatigue range. As far as an extrapolation of the relationship in the high cycle fatigue range is needed this will be achieved in practice by a bend in the P-S-N curve and a new, again linear, double logarithmic line continuing with a now shallower slope into the high cycle region. This is the usual treatment for design purposes in practice.

If a monotonic analytical relationship for the whole range of cycles to failure is demanded, from the static limit strength up to the endurance limit, this may be achieved by a non-linear P-S-N curve. Two basic types of four-parametric curves have been proposed. Weibull modified an older proposal by Palmgren, based again on a function by Stromeyer, presented in the following general form

$$\sigma = (\sigma_z - \sigma_D) \cdot \left(\frac{N}{B} + 1 \right)^{-a} + \sigma_D \quad (7)$$

with
 σ_z : ultimate strength
 σ_D : endurance limit
 B : time parameter
 a : form parameter

From $d^2(\sigma)/d(\log N)^2 = 0$ we get the transition point of the curve $N_i = B/a$ further we have the following relationships for the shape of the curve: $d\sigma/d\log N = 0$ and $d^2(\sigma)/d(\log N)^2 < 0$ for $N < N_i$ and $d^2(\sigma)/d(\log N)^2 > 0$ for $N > N_i$ when the function is plotted in linear-logarithmic co-ordinates.

The second curve type, also a four-parametric relationship, was proposed by Stüssi as follows

$$\sigma = \frac{\sigma_z + bN^a \cdot \sigma_D}{1 + bN^a \cdot \sigma_D} \quad (8)$$

The four parameters of the equations can be calculated by a multiparametric non-linear regression analysis. The above expressions as functions of the four parameters have a specific value (by assuming initial values for each of the four parameters) which shows a deviation v_i for each available experimental observation M_i . By minimisation of these deviations

$$\sum_{i=1}^n v_i^2 = \text{Min} \quad (9)$$

we get a system of n equations out of which the four unknown parameters may be calculated. Details of the procedure are given in literature or in a respective calculation procedure within the Aluminium Data Bank.

Some Problems of Data Analysis in Practice

The basic method of analysis described has been the calculation of the best-fit regression curve of $\log N$ on $\log \Delta \sigma$ (appropriate in the case of welded joints) to fatigue data by the method of least squares. The S-N curves have been assumed to be linear plotted in log-log coordinates, where the stress is the nominal applied stress range in the vicinity of the weld detail. The standard deviation of $\log N$ about the regression line is calculated and used to establish confidence limits based on the assumption that the data conform to log-normal distribution, **Figure 2401.03.01**. Although the confidence limits are strictly hyperbolae which are nearest to the mean regression line at the mean value of $\log \Delta \sigma$ covered by the data, for convenience it is assumed to be sufficiently accurate to assume that they are equivalent to tangents drawn to the hyperbolic confidence limits parallel to the mean regression line. The lower 95% confidence limit, which is approximately two standard deviations below the mean regression line, is chosen as a suitable basis for design - it depends on the respective recommendations, whether additional material safety factors may be defined or not. It corresponds theoretically to a probability of failure of 2.5% or to a probability of survival of 97.5%.

We would like to draw attention to the fact that in the above context the term "confidence limits" denotes a scatter band of the test data of a given probability of occurrence. It is not understood in the sense of confidence limit for a certain fractile of the statistical analysis expressing the uncertainty associated with the calculation of this fractile because of the estimation performed out of a specific sample size only (i.e. a limited number of observations).

When applying regression analysis it is important to ensure that all the data considered can be expected to correspond to the assumed relationship. In the case of fatigue data this is relevant, because, although the linear $\log \Delta \sigma$ - $\log N$ relationship will apply over a wide range of stress-life conditions, deviations towards horizontal lines on the S-N diagrams may occur in the high stress-low cycle regime, when the maximum stress exceeds yield, and at low stresses as the stress range approaches the fatigue limit. In practice, deviations in the low-cycle regime are easily identified because the yield strength of the material is known. However, the fatigue limit is a property of the joint tested and its value not always obvious. Appropriate procedures were outlined in **Figure 2401.03.05**, **Figure 2401.03.06** and **Figure 2401.03.07**.

Another problem mentioned when analysing fatigue data from several sources is the question of whether or not all the data can be assumed to belong to the same population. This is found to be a problem for some sets of data, even when they are obtained from geometrically similar joints under the same loading conditions. An all-data analysis produces a best-fit S-N curve whose slope is incorrect in that it is quite different from the slope indicated by the separate analysis of the individual sets of data. The fact that data belong to different populations can be demonstrated by a plot as in **Figure 2401.03.04**. If nevertheless a common analysis has to be undertaken, because of lack of sufficient appropriate data, the following procedure may be adopted. Calculate the best-fit S-N curve for each set of data, calculate the median value of their slopes and then assume that this is the slope of the S-N curve for all the data together, the mean S-N curve passing through the centre of gravity of the data points.

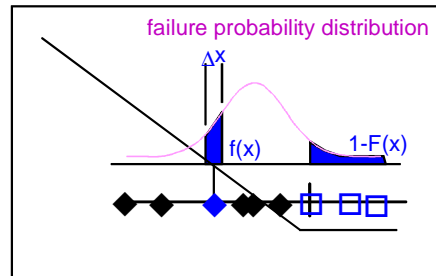
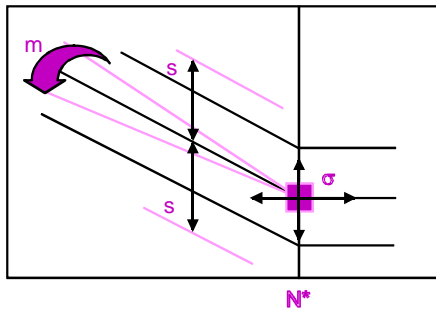
Aluminium weldments seem to be more susceptible to problems as outlined. They are susceptible to mean stress and the level of residual stress due to welding, reflecting the greater sensitivity of crack growth to mean stress in aluminium alloys. Variations in the level of residual stress in a given type of joint from one investigation to the next could arise as a result of variations in the welding conditions. The effect of differences in applied mean stress related to the residual stress level could be to influence both the slope and position of the S-N slope. In practice, welded joints containing high tensile residual stresses are of interest to the designer of a welded structure. Thus test data obtained from relatively large specimens, preferably having the detail incorporated in a structural element such as a beam, or from specimens tested under a high tensile mean stress are of special interest - **Lecture 2402.03** demonstrates the significance of such information for design purposes through the structural details investigated and evaluated. In order to utilise other data, it may be necessary to correct the S-N curves fitted to them to allow for the absence of high tensile residual stresses, see here also chapter 6 on residual stress influence and fatigue strength correction factors. S-N curves for higher R values show steeper slopes, the curves for different R values meeting close to material yield strength in the range between 10^4 and 10^5 cycles.

The validity of grouping together data sets for a given joint type or the general validity of test results depends on further factors, like plate thickness, alignment of the joint and test environment. Plate thickness not only affects the level of residual stresses, it is also important in the case of joints which fail by fatigue crack growth from a surface stress concentration such as a weld toe. There can be a significant reduction in fatigue strength with increase in plate thickness, see respective provisions for plate thickness above 25 mm in the recommendations in **Lecture 2402.01** and **2402.03**. Misalignment is difficult to avoid in small specimens, particularly in the case of transverse butt welds. Its effect is to introduce secondary bending stresses when the specimen is tested axially such that the stress near the weld may be quite different from the nominal stress based on load/plate cross-sectional area, that is the stress normally used to express fatigue test results. The presence of misalignment may be a major cause of the relatively very wide scatter in transverse butt and fillet welds and, of course, the applicability of laboratory (small specimen) test results to real structures operating in potentially corrosive conditions is questionable.

A technique which could be used to deal with the above problems, both from the point of view of deciding whether or not different data sets could be regarded as belonging to the same population and the analysis of several data sets with an S-N curve whose apparent slope differs from the individual slopes, is the maximum likelihood method, **Figure 2401.03.10**. The method also is able to handle results from unbroken test specimens (run-outs). Run-outs arise either because the test specimen endures a predetermined life without failing or because testing is stopped when some special failure criterion is satisfied, for example the presence of a detectable crack. The former is particularly relevant since frequently, especially in the past, it was quite common to stop tests at only $2 \cdot 10^6$ cycles on the basis (probably stemming from test results for plain unwelded specimens) that the fatigue limit would correspond to such an endurance. In practice, test data suggest that an endurance limit of around $5 \cdot 10^6$ cycles would be more appropriate. For aluminium such an adjustment has been taken into account in design recommendations, see **Lecture 2402.03**.

Maximum-Likelihood-Method

P-S-N curve estimation



Through curve parameter (life limit N^* , fatigue endurance, slope and scatter) variation determination of max SUP.
Result: maximum-likelihood P-S-N/ curve

$$SUP = \prod [\Delta x f(x) (1 - F(x))]$$

failures probability of failure
run-outs probability of failure

Source: D. Kosteas, TUM



Maximum-Likelihood-Method

2401.03.10

Closing these remarks we would like to remind about the comments made earlier on the merits but also on the problems associated with the application of statistical methods in evaluating and comparing fatigue test data. It seems that in many cases where we lack visual apparentness of facts in fatigue data this cannot be substituted or provided by mathematical assumptions. An alternative approach on the basis of fracture mechanics and crack growth analysis using basic information from similar tests can be recommended, especially in view of the general problem of scatter.

2401.04 Load Spectra and Damage Accumulation

- Service behaviour
- Time-dependent loads
- Spectrum definition and cycle counting
- The rain-flow cycle counting method
- The service behaviour fatigue test
- Analytical life estimation and damage accumulation
- The Palmgren-Miner linear damage accumulation hypothesis
- Service behaviour assessment

Service Behaviour

Structural components in service are exposed to a more or less random sequence of loading of variable amplitude and frequency. Even when maximum values of these loading inducing stresses up to 2 or 3 times the fatigue endurance limit appear in relatively small numbers, somewhat lower amplitudes may reach considerable number within the lifetime of a structure and together with irreversible deformations or local damage, like flaws, cracks, etc., reduce the carrying capacity and the design life.

It is the goal of a fatigue service behaviour analysis to assess through an appropriate procedure the probability for a given structure under a given loading to reach without failure or extensive necessary repair the demanded design life.

Components in many structural engineering applications, esp. in metal structures in civil engineering, are only rarely assessed under the principles of a service behaviour analysis - and this situation is still reflected in the current recommendations for fatigue design. With increasing service lives and the use of new or special materials and manufacturing methods this approach is not sufficient. Besides, service loading conditions will in general be more favourable than the constant amplitude loading (Wöhler-test) forming the basis of fatigue endurance investigations. Service conditions on the other hand may change. Several applications in land and sea transportation, cranes or even some bridge components demand for such a service behaviour assessment accounting for a variable loading sequence and damage accumulation.

Principles of light structural design cannot be satisfied if a component is designed merely against the maximum value of a load or stress spectrum. The accurate definition of the latter for a specific application still poses great difficulties and it is on the loading side of the analysis that the greatest uncertainties emerge. Only with a reliable description of the loading can suitable damage accumulation hypotheses be developed and applied to the calculation of fatigue life.

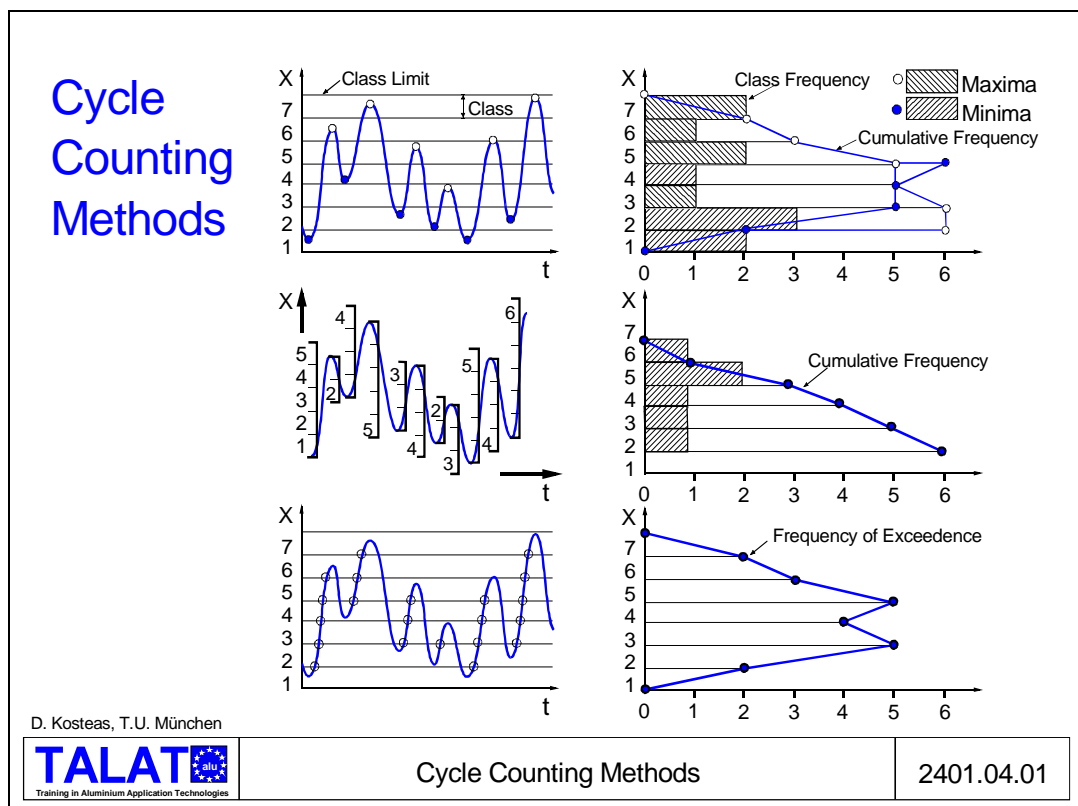
Time Dependent Loads

Loads on a structural component depend, first, on the usage of the structure (steering, accelerating or established procedure forces) and, second, act on the structure out of its environment (wind, wave motion, track smoothness) and will generally be time dependent. They may be deterministic or random in nature. In the first case (as for instance periodic or non-periodic load like the influence of an impact load or the thermal deformation) a mathematical relationship allows a definite expression of the value of the characteristic quantity at any time. In the second case (and here we encounter most mechanical loads) values of the characteristic quantity can be estimated only through a time sequence measurement. Naturally they are unique and not reproducible and consequently the estimation can be expressed only with a certain probability.

A stochastic process is defined as non-stationary or stationary depending on whether its statistical characteristic values are varying or not with time. This can not be identified in practice seldom though since service load measurements are performed only once as a rule.

Spectrum Definition and Cycle Counting

A load spectrum includes all necessary information on magnitude and frequency of service loads, possibly also about the loading sequence. For each structural detail a respective stress spectrum results expressing the frequency of occurrence of a characteristic value such as the stress amplitude, stress range or maximum stress.

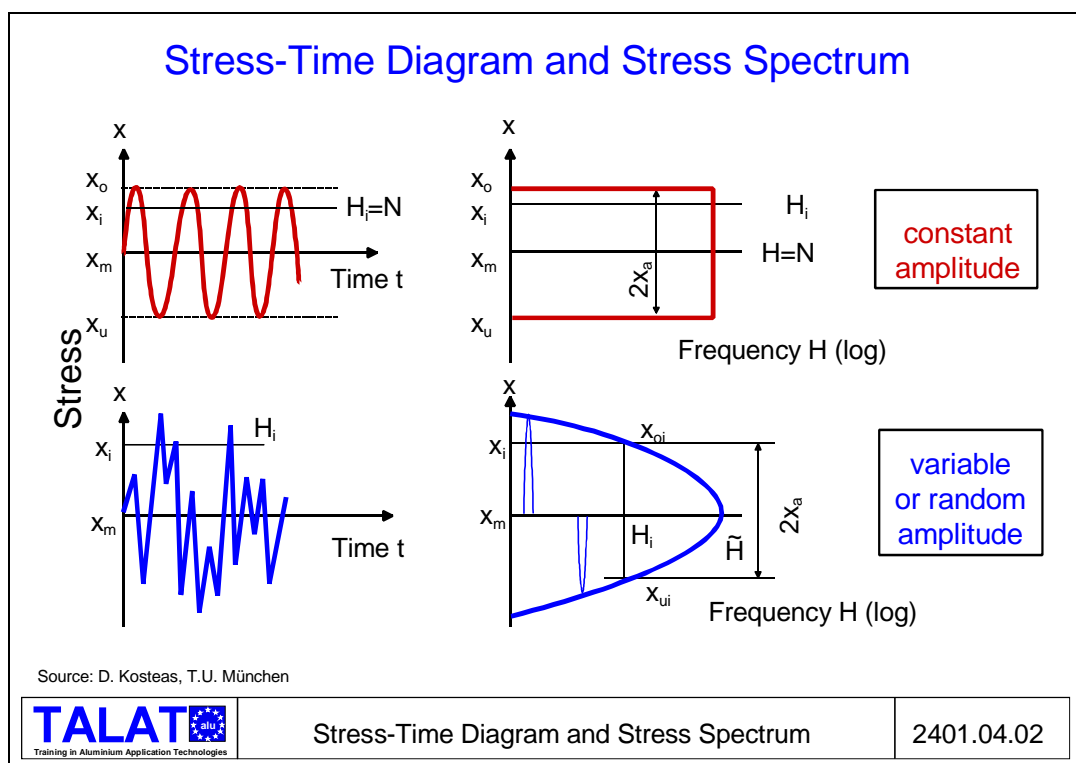


The spectrum is derived from measurements of the characteristic value over the observed cycles for a specific time period. Three basic counting methods can be applied as demonstrated also in **Figure 2401.04.01**:

- the measured value reaches a turning point, maximum or minimum,
- the measured value encompasses a range, between a minimum and the following maximum or vice versa,
- the measured value touches or surpasses a defined class limit in ascending or descending order.

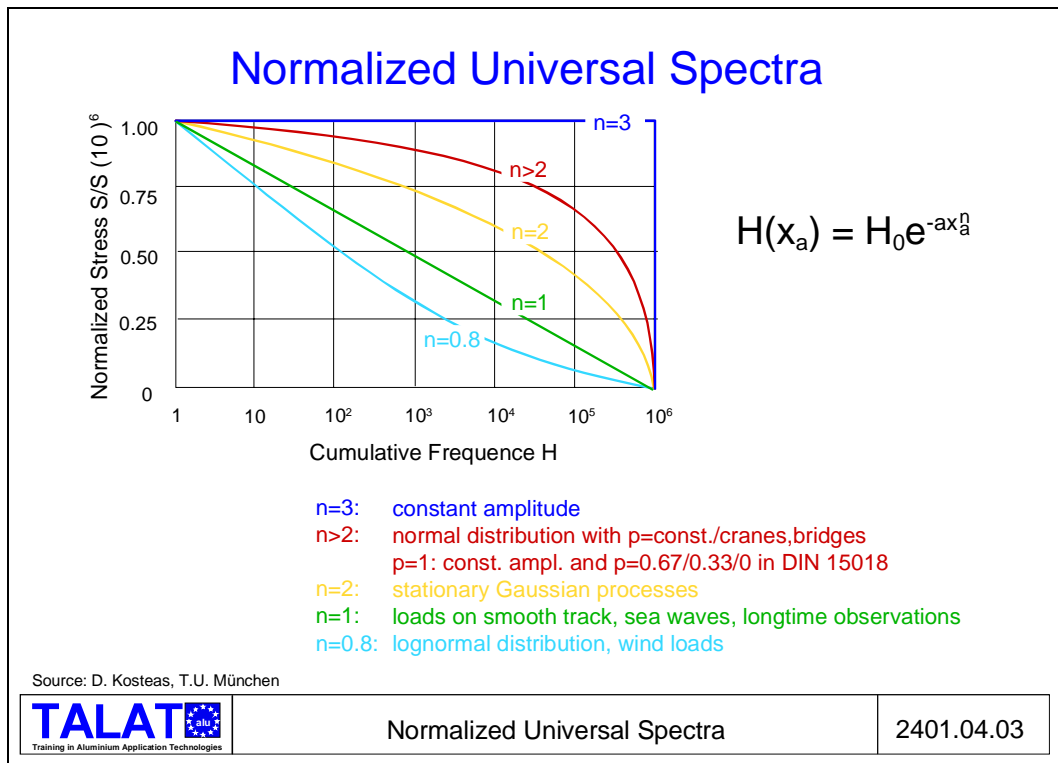
The graphic depiction gives the so-called histogram, which turns into the probability distribution function for an extremely large number of observations. This frequency distribution is shown usually in a linear (stress) - logarithmic (cumulative frequency) distribution and is the so called spectrum, **Figure 2401.04.02**.

An estimate of the magnitude of the difference between the original, measured stress-time function and the derived spectrum is given by $i=H_0/H_1$, where H_0 is the number of passes through zero (or the reference value) and H_1 the number of turning points (maxima, minima). For random stress-time functions we have $0 < i \leq 1$ and for in the above manner derived frequency distributions $i=1$.



A normalisation, **Figure 2401.04.03**, allows the comparison between spectra of different maximum values or absolute cumulative frequency. In many cases in structural engineering a spectrum cumulative frequency of 10^6 is used; the maximum value with a frequency of 1 in 10^6 is designated $x_{a, 10^6}$.

Experience shows that most of the observed and measured stress-time functions follow a few basic statistical distributions. This is very important for the calculation and adaptation of fatigue test results to cases with different loading conditions through the so-called damage accumulation hypotheses, a concept which poses a number of difficulties.



Following a suggestion by Gassner/Griese/Haibach the mathematical expression formulated by Hanke $H(x_a) = H_0 \cdot \exp^{-ax_a^n}$ describes five such basic types, **Figure 2401.04.03**. Type (a): for $n<2$ represents a spectrum with constant amplitude. Type (b): for $n>2$ is typical for spectra in crane and bridge structures, which can be seen as Gaussian normal distributions with a constant part p . Here again $p=1$ would refer to the constant amplitude, and values of $p=2/3, 1/3, 0$ have been suggested for respective cases in the German national standard for cranes DIN 15018. Type (c): for $n=2$ is typical for stationary Gaussian processes. Type (d): for $n=1$ is the so-called linear distribution as it appears with a straight line in the linear-log diagram and is characteristic for loading conditions due to track smoothness or sea wave motions and for long observation periods. Type (e): for $n\approx 0.8$ is typical for wind loads and follows the shape of a logarithmic normal distribution.

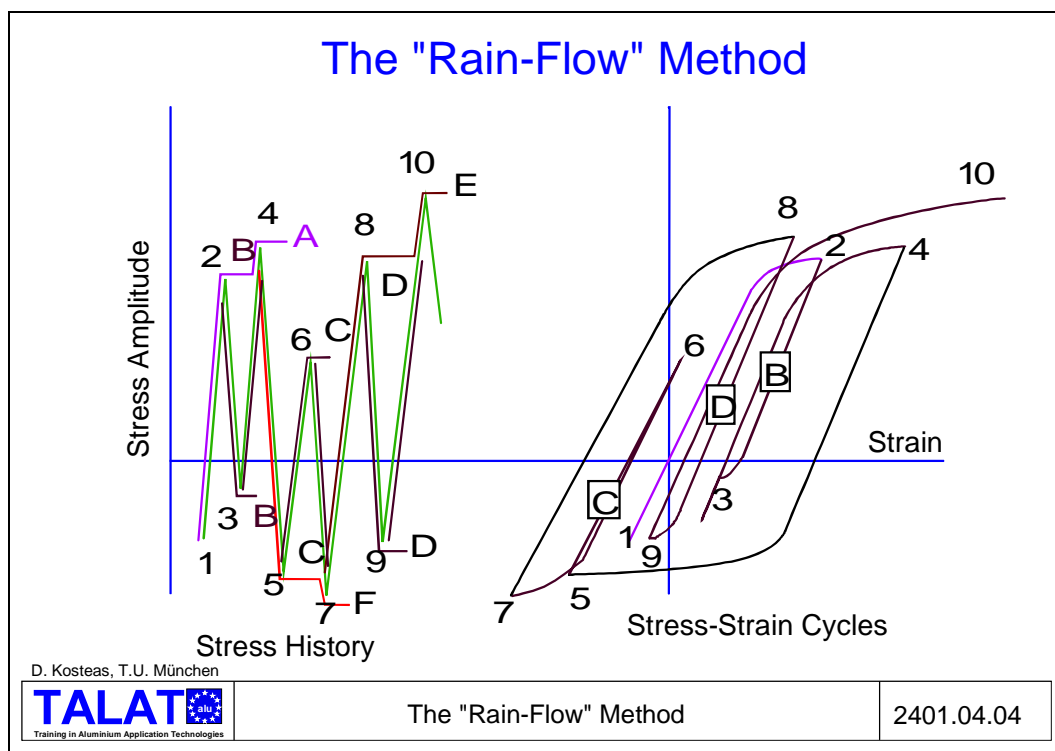
The frequency distribution allows not only the comparison with other distributions but also enables the extrapolation beyond the actual measurement period to other, longer ones and consequently up to the design life of the structure. Two conditions must be fulfilled: (a) the measured event must be representative for the whole design life, the sample measured must be adequate, and the different service conditions represented with their respective relative frequencies; (b) an extrapolated maximum value must still be physically feasible. The universal spectra mentioned are exponential functions which for infinite observation periods furnish infinitely large x_a values. Extrapolations are undertaken for reasons of simplicity on such scales that let the frequency distribution

(spectrum) appear as a straight line. Measurement data exhibit considerable scatter and such extrapolations may become questionable. Preferable are methods based on extreme value distributions that allow the estimation of such maximum values through statistical tools adapted to engineering [3].

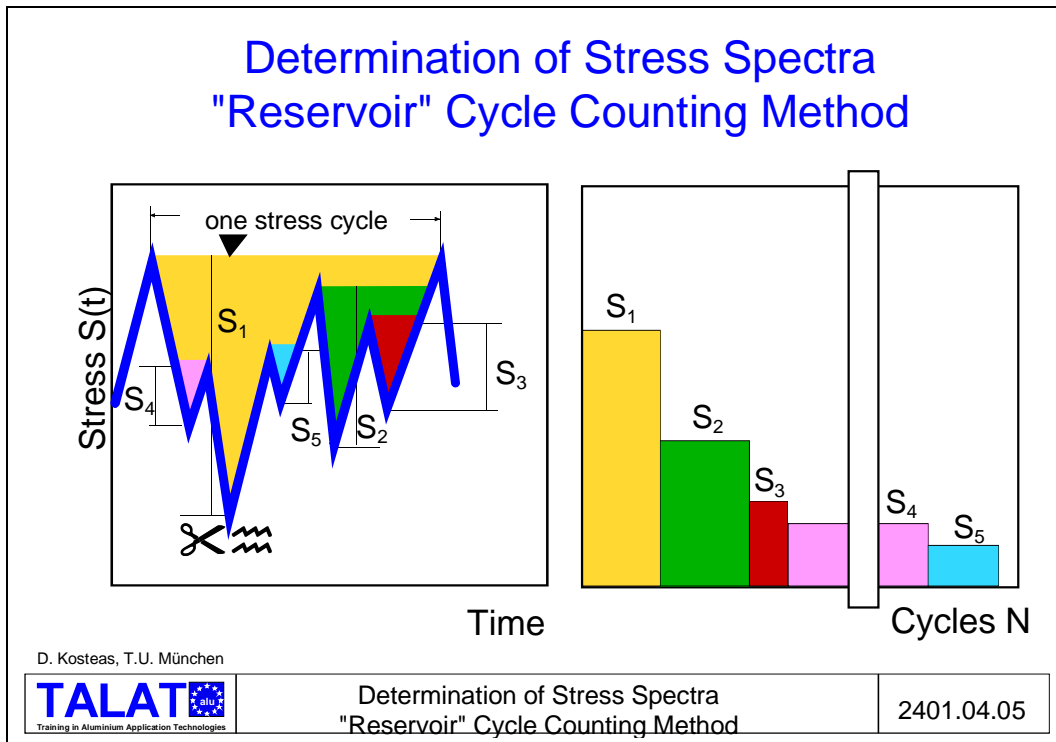
Two-parametric counting methods register two consecutive characteristics in an effort to include information about the loading sequence as well. In fatigue two methods are of interest, the "range-mean counting" and the most frequently used "rain-flow cycle counting" (see next paragraph).

The Rain-Flow Cycle Counting Method

The method is based on the forming and counting of full cycles out of the original amplitude-time diagram. Practically this is done through registering those stress amplitude parts of the stress amplitude-time diagram over which a rain drop would flow as indicated in **Figure 2401.04.04**.



The same result is obtained by the better comprehensible "reservoir method", **Figure 2401.04.05**. The stress-time diagram is filled with water like a reservoir, the water is let out at the lowest point and the water column height gives the respective cycle with a range $\Delta\sigma_1$. The procedure is repeated at the next lower point and so on. The stress ranges are collected in classes and result in the cumulative frequency diagram, the stress spectrum. The counting method does not account for mean stresses or the R-ratio but this does not present a problem in life estimations in practice since these are based on an assessment of stress ranges.



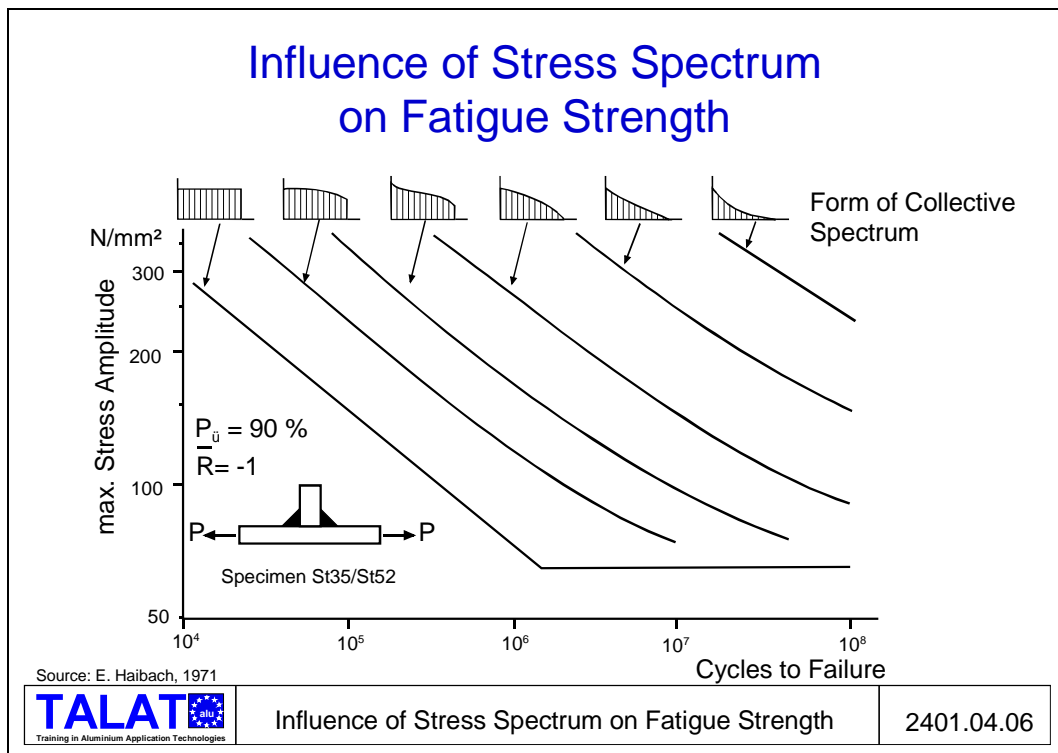
The Service Behaviour Fatigue Test

It is the goal of fatigue tests with an appropriate spectrum loading to establish such strength-life limit curves for structural components that may lead to generalised design criteria in practical cases, **Figure 2401.04.06**. Such tests are rather costly and time consuming and as such they will be realised on a greater scale only in products manufactured in larger numbers. In aircraft as well as in several applications of ground transport vehicles where random loading sequences are simulated during the test procedure such tests have their justification. In all other cases the assumption of a damage accumulation hypothesis in relation to fatigue data from constant amplitude tests will be the rule.

Depending on the extent of idealisation of the original loading events in service we may distinguish between different spectra and tests. Results of these tests with variable amplitudes can be analysed in a way similar to the one for constant amplitude tests. They may be characterised by analogous expressions for the maximum or minimum amplitudes, ranges or ratios and the stress-life curve

$$R^* = \frac{\min \sigma^*}{\max \sigma^*} ; \Delta \sigma^* = \sigma_m^* - \sigma_a^* ; \Delta \sigma^* = \Delta \sigma_{10^6}^* \cdot \left(\frac{10^6}{N^*} \right)^{-m^*}$$

whereby the quantity N^* indicates the total number of cycles in the spectrum, often assumed to $N^* = 10^6$.

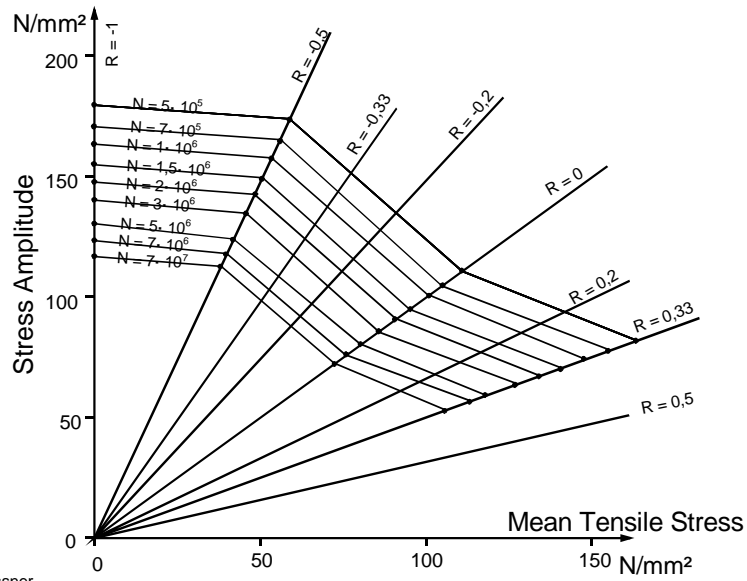


A characteristic example of such analyses is given in **Figure 2401.04.07** for flat notched (stress concentration factor $\alpha=3.6$) specimens of AlCuMg2 alloy under axial loading [3]. The testing frequency was variable and the lines are for a probability of survival 50%. **Figure 2401.04.07** shows the results for a spectrum form of the normal distribution.

Relationships between the lives realised in constant and variable amplitude tests can be established only empirically because of the very complicated damage mechanisms. Schütz has performed comprehensive investigations and the two diagrams in **Figure 2401.04.08**. **Figure 2401.04.08** shows the results for aluminium alloy AlCuMg2 (AA2024) [3]. Loading was axial and the probability of survival 50%.

Fatigue Behaviour Diagram

AlCuMg2 Spectrum Behaviour: Normal Distribution



Source: Schütz, Gassner

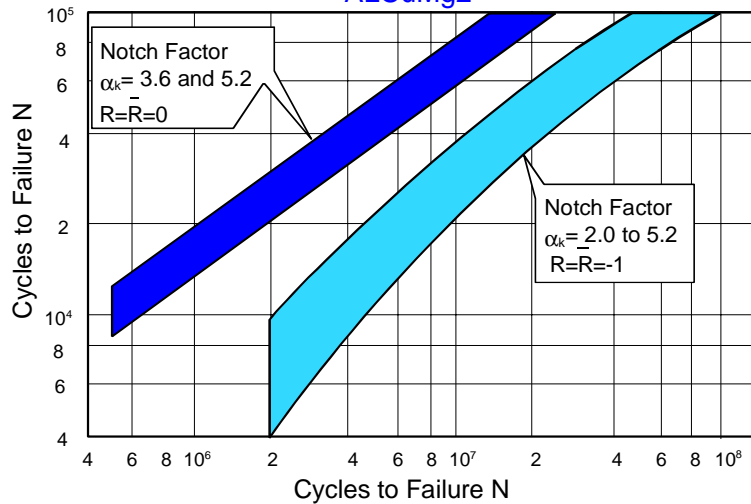


Fatigue Behaviour Diagram

2401.04.07

Fatigue Strength for Constant and Variable Amplitude (Normal) Loads

AlCuMg2



Fatigue Strength for Constant and Variable Amplitude (Normal) Loads

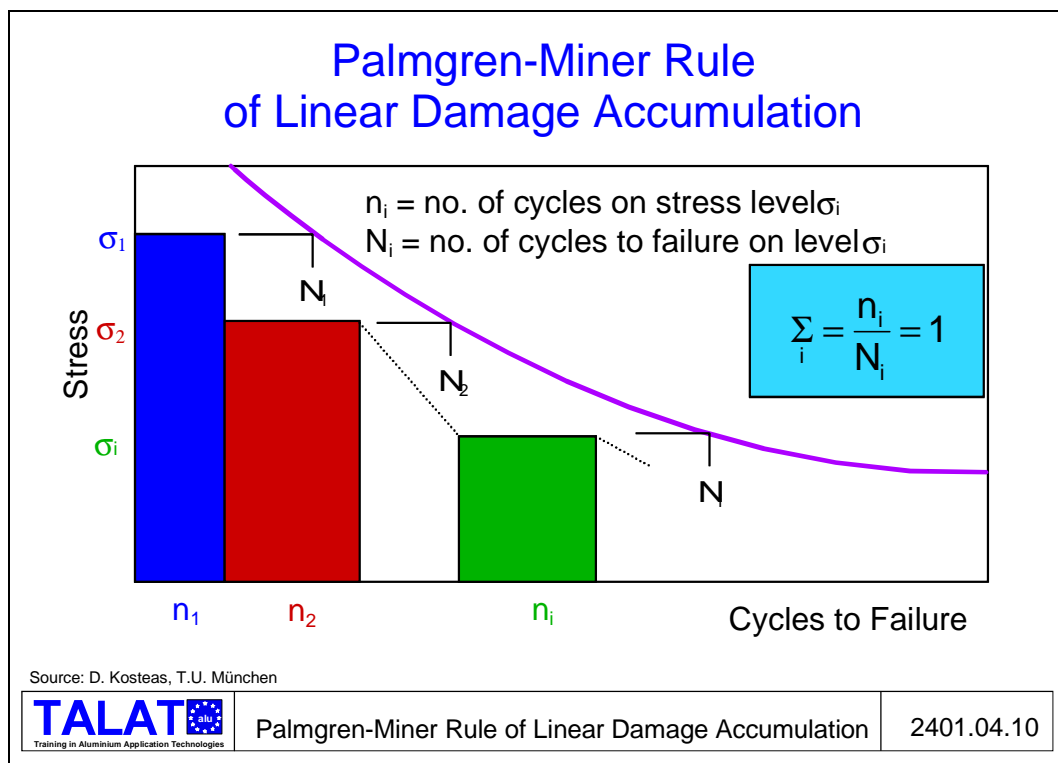
2401.04.08

The Palmgren-Miner Linear Damage Accumulation Hypothesis

The hypothesis rests on two basic assumptions: (a) the damage D in a constant amplitude test grows linearly with the number of cycles n until these reach the number of cycles to failure N , for which the damage D reaches the value of one; (b) for a loading sequence with variable amplitudes the partial damage D_i on the amplitude level i can be summed with other partial damages and failure (fracture) will appear when the sum of partial damages reaches unity, i.e.

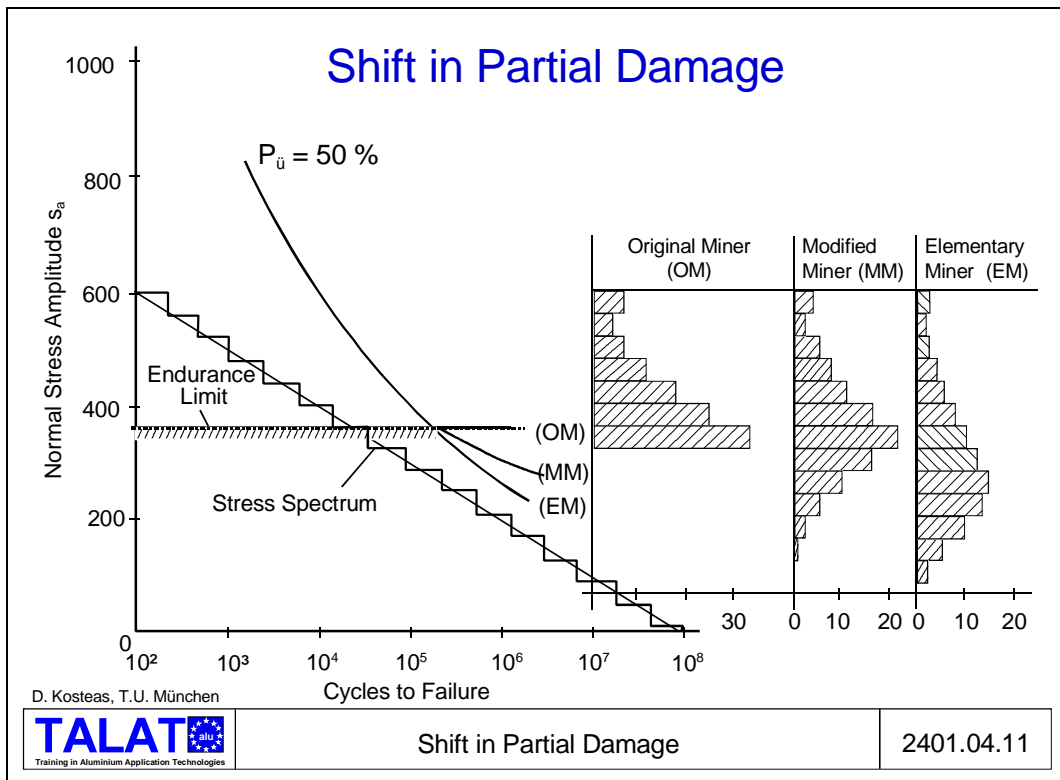
$$D = \sum_i D = \sum_i \left(\frac{n_i}{N_i} \right) = 1 \text{ for fracture}$$

Schematically the application of the linear damage accumulation hypothesis is shown in **Figure 2401.04.10**.



Already Miner himself had warned against a general application of the linear damage accumulation rule. It should be taken into account that the linearity itself does not strictly exist, the sequence of loading and local residual stresses are not considered, and, theoretically, stresses below the fatigue endurance limit of the constant amplitude S-N line do not contribute to the damage summation.

Numerous suggestions have been made to improve the original hypothesis in the above mentioned points by changing the constant amplitude S-N line, esp. through various extrapolations of it below the constant amplitude cut-off limit, **Figure 2401.04.11**.



It is really a whim of statistics, if the mean of fatigue life estimations for stochastic loading processes evaluated with the Palmgren-Miner rule compared to respective test results is approximately at the value of one. It can be formulated that the ratio of actual to estimated life may vary between 0.2 and 6, or in other words as 1:30 (**Figure 2401.04.12**). An engineer should try to visualise this statement: a fatigue life estimation on the basis of the Palmgren-Miner hypothesis can be either five times lower or six times higher than the actual value! On the other hand too conservative statements, such as generally limiting the allowable damage sum to 0.3 for instance would result in uneconomical design.

Life Estimation with the Palmgren-Miner Rule

in practice the damage sum varies from 0.2 to 6

A fatigue life estimation on the basis of Palmgren-Miner can be either
five times lower or six times higher
than the actual value

Visualize !

Too conservative limits of the allowable damage sum would, on the other hand, result in uneconomical design

Source: D. Kosteas, T.U. München

 <small>Training in Aluminium Application Technologies</small>	Life Estimation with the Palmgren-Miner Rule	2401.04.12
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Many of the observed uncertainties associated with the linear damage hypothesis are not of accidental nature. Some tendencies have been observed (**Figure 2401.04.13**). An estimation on the (1) unsafe side will result when there are

- (a) large fluctuations of the basic or mean stresses,
- (b) an S-N constant amplitude curve for bending stresses is used,
- (c) stress-time function with a large number of cycles below the endurance limit,
- (d) treatments introducing compressive residual stresses, i.e. the use of respective constant amplitude S-N curves,
- (e) higher temperatures.

Estimated values will tend to the (2) safe side when

- (a) stress-time functions with positive basic or mean stresses are used,
- (b) measures introducing tensile residual stresses are taken,
- (c) eyebars are assessed,
- (d) components with compressive residual stresses are assessed as far as the respective constant amplitude S-N curve has been established on specimens without significant compressive residual stresses.

Observed Tendencies with the Palmgren-Miner Rule

<p>Results on the unsafe side, when</p>	<ul style="list-style-type: none"> - large fluctuations of mean stress - S/N for constant bending used - large no. cycles below endurance limit - treatments for compressive resid. stresses - higher temperatures
<p>Results on the safe side, when</p>	<ul style="list-style-type: none"> - stress history with positive mean stresses - treatments for tensile residual stresses - eyebars assessed - S/N for constant amplitude without significant compressive residual stresses applied to components with compressive residual stresses


Source: D. Kosteas, T.U. München

TALAT <small>Training in Aluminium Application Technologies</small>	Observed Tendencies with the Palmgren-Miner Rule	2401.04.13
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A generally larger scatter of the estimations will result if there is a change in the site of fracture as a consequence of a change in the stress intensity, or if there is a variation (enhancement or reduction) of residual stresses which is not accounted for in the respective constant amplitude S-N curve, and, finally, if a relocation of the force transfer is encountered during the lifetime of the component, as can be the case with joints.

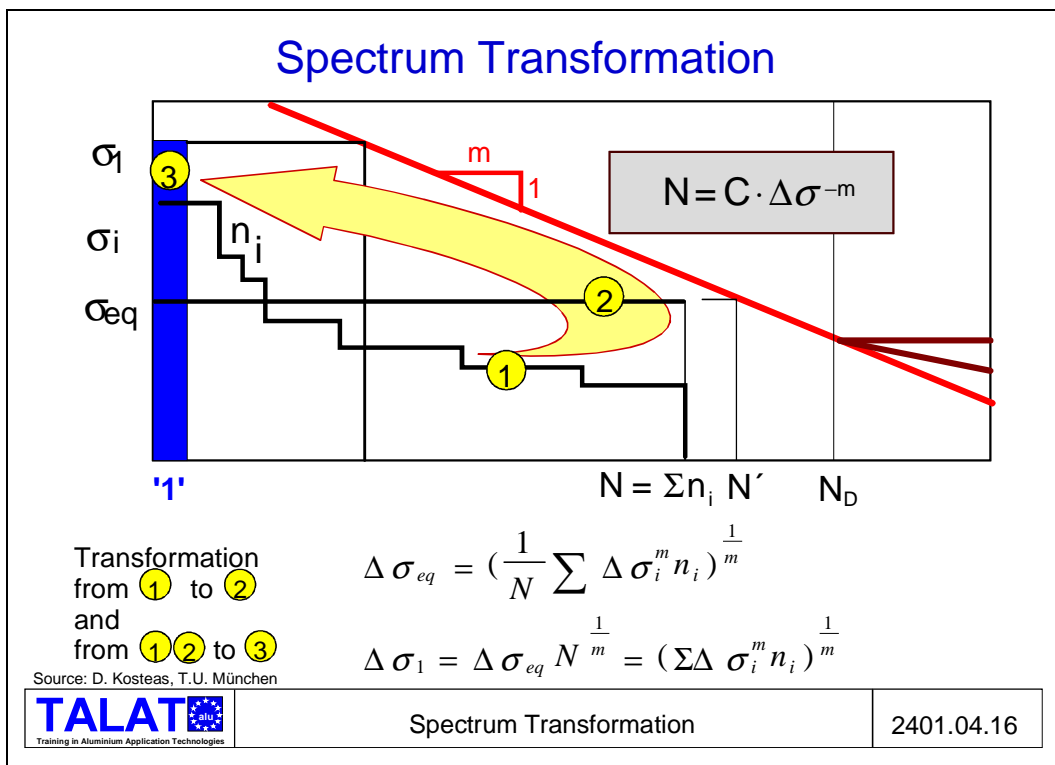
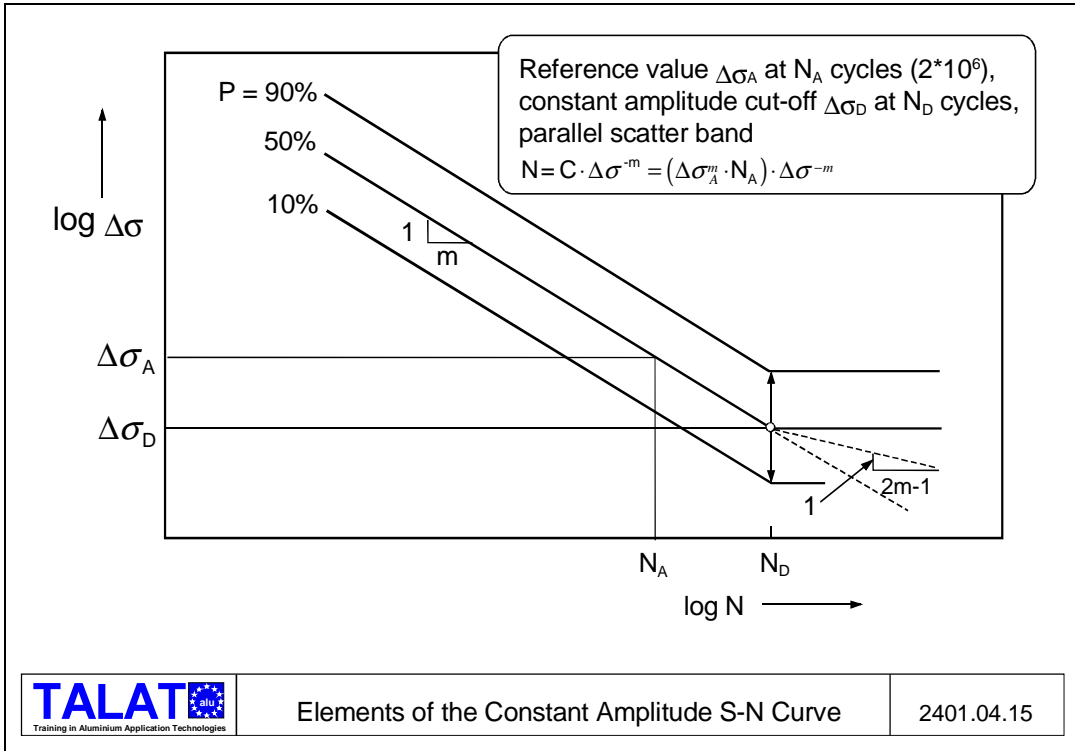
As a final fact it should be remembered that the quality of an estimation by means of the linear damage accumulation hypothesis (i.e. the scatter) cannot be influenced significantly by observing the above points. This experience leads to the fact that the quality of

the damage accumulation rule is characterised primarily not by the fact whether summation results near unity are observed, but rather by the fact whether the observed scatter is sufficiently small (**Figure 2401.04.14**).

<h2 style="color: blue;">Experience with the Palmgren-Miner Rule</h2>		
<p>1. Rather large scatter in estimates, if</p> <ul style="list-style-type: none">- change of SIF resulting in change of site of fracture- variation of resid. stresses not accounted for in the respective S/N curve- relocation of force transfer during lifetime (joints !)		
<p>2. Quality of estimate cannot be influenced significantly</p> <p>the quality of the rule is characterised primarily by the fact whether observed scatter is sufficiently small and <u>not</u> whether summation results near unity are observed</p>		
<small>Source: D. Kosteas, T.U. München</small>		
	Experience with the Palmgren-Miner Rule	2401.04.14

Service Behaviour Assessment

Many of the elements already described concerning the actual fatigue testing of specimens or structural components, their analysis and evaluation, influencing parameters, as well as the above information on variable amplitude loading constitute parts of the assessment procedure under service conditions. The recent recommendations for aluminium constructions follow to a certain extent these procedures, especially in relation to spectrum loading and its evaluation through reference to the established constant amplitude S-N curves of the different structural details and by means of the linear damage accumulation. **Figure 2401.04.15** and **Figure 2401.04.16** give the elements of the constant amplitude curve and the calculation procedure of the "equivalent stress" based on Palmgren-Miner. **Figure 2401.04.16** illustrates the load spectrum transformation for the service behaviour assessment depending on the application. The original spectrum ❶ may be transformed by means of the linear damage accumulation hypothesis to either an equivalent stress ❷ with the same total number of cycles as the original spectrum or to an equivalent stress ❸ for a "single loading event" (as for instance the passage of a multi-axle vehicle, train etc.).



Literature

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2401.05 Local Stress Concepts and Fatigue

- Analytical relationship between strain and fatigue life
- Notch theory concept
- The strain-life diagram

Analytical Relationship between Strain and Fatigue Life

Describing predictive theories of fatigue in **Lecture 2401.02** we have mentioned the relationship between strain range $\Delta\varepsilon$ and the number of cycles to failure, the semiempirical Manson-Coffin law. Especially in the so-called low-cycle-fatigue range where plastic strain ranges contribute significantly to the strain vs. fatigue life relationship, such an approach describes the phenomenon in a satisfactory way. As already mentioned the Manson-Coffin relationship, or as modified by Morrow, states that

$$\frac{\Delta\varepsilon_t}{2} = \frac{\Delta\varepsilon_{el}}{2} + \frac{\Delta\varepsilon_{pl}}{2} \quad (1)$$

This can be transformed as follows

$$\frac{\Delta\varepsilon_t}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} \quad (2)$$

and taking into account the log-log relation after Basquin $\Delta\sigma = C \cdot N_f^B$

$$\frac{\Delta\varepsilon_t}{2} = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \quad (3)$$

where

$\Delta\varepsilon_t$	true strain at notch
$\Delta\varepsilon_{el}$	elastic part of true strain
$\Delta\varepsilon_{pl}$	plastic part of true strain
$\Delta\sigma$	effective stress at notch
E	elastic modulus of the cyclic stress-strain-curve
K'	cyclic stress coefficient
n'	cyclic hardening exponent
σ'_f	true ultimate strength
b	fatigue strength vs. life exponent
ε'_f	fatigue ductility coefficient
c	fatigue ductility exponent
$2N_f$	reversals to failure = total cycles N to failure

The goal is the estimation of $N = 2N_f$ cycles to failure for a given notch strain condition expressed through $\Delta\varepsilon_t$. In practical applications we are interested in a simple relation

between nominal stresses on the structural component and cycles to failure. The connection between nominal design stresses and the local notch stresses or strains can be established through the analytical notch theory concept of Neuber, as explained below.

All other parameters as given in the above strain vs. life relationship are to be estimated from experimental data, i.e. cyclic stress-strain relationships expressed analytically through the Ramberg-Osgood formula

$$\varepsilon = \frac{\sigma}{E} + 0.002 \left(\frac{\sigma}{R_{p,0.2}} \right)^{n'} \quad (4)$$

with

$R_{p,0.2}$ = yield stress at 0.2% plastic strain in cyclic stress-strain relation

In case of spectrum loading respective damage accumulation theories, see **Lecture 2401.02**, may be used.

Notch Theory Concept

Defining local stress or strain as σ and ε and the respective nominal values as S and e we define the theoretical stress or strain intensity factors

$$K_{\sigma} = \frac{\sigma}{S} \quad \text{and} \quad K_{\varepsilon} = \frac{\varepsilon_t}{e_t} \quad (5)$$

Neuber defines the theoretical intensity factor as

$$K_t = (K_{\sigma} \cdot K_{\varepsilon})^{1/2} \quad (6)$$

and Topper assumes a similar relationship for the case of cyclic loading substituting the stress and strain values with the respective stress and strain ranges whereby the cyclic notch factor K_f is introduced

$$K_f = \left(\frac{\Delta\sigma \cdot \Delta\varepsilon_t}{\Delta S \cdot \Delta e_t} \right)^{\frac{1}{2}} \quad (7)$$

This last formula may be transformed into

$$\Delta\sigma \cdot \Delta\varepsilon_t = \frac{1}{E} (K_f \cdot \Delta S)^2 \quad (8)$$

which gives us the possibility to determine the product $\Delta\sigma \cdot \Delta\varepsilon_t$ on the condition that on the right side of the formula is determinable. The latter may be achieved by estimating the only unknown, K_f , from an empirical relationship such as given by Peterson

$$K_f = 1 + \frac{K_t - 1}{1 + \frac{a}{r}} \quad (9)$$

where

a is a material constant with an approximate value of a = 0.5 mm for aluminium alloys according to Peterson
r is the notch radius

In case of fatigue cracks the so called 'worst notch case' is assumed, i.e. a maximum value for K_f has to be estimated, which will be produced for a certain relation of the material constant a, the geometrical notch form, and the critical notch radius. In the most common case of an elliptical crack shape $r_{crit} = a$ and $\max K_f = 1 + 2(t/a)^{1/2}$ with t = plate thickness.

The Strain-Life Diagram

With the help of a cyclic stress-strain curve for the material observed the pair of values $\Delta\sigma$ and $\Delta\varepsilon_t$ may be calculated according to the now known value of their product as given with equation (8). Finally with the calculated $\Delta\varepsilon_t$ value the desired fatigue life to failure according to equation (3) can be determined and the respective ε -N curve can be constructed.

According to a proposal by Smith-Watson-Topper the quantity $(\sigma_a \cdot \varepsilon_a \cdot E)^{1/2}$ may be used to characterize the fatigue behaviour of a material in a respective diagram. Through equation (2) and (3) a new relation is established

$$\sqrt{\sigma_a \cdot \varepsilon_a \cdot E} = \sqrt{\sigma_f'^2 \cdot (2N_f)^2 + E \cdot \sigma_f' \cdot \varepsilon_f' \cdot (2N_f)^{b+c}} \quad (10)$$

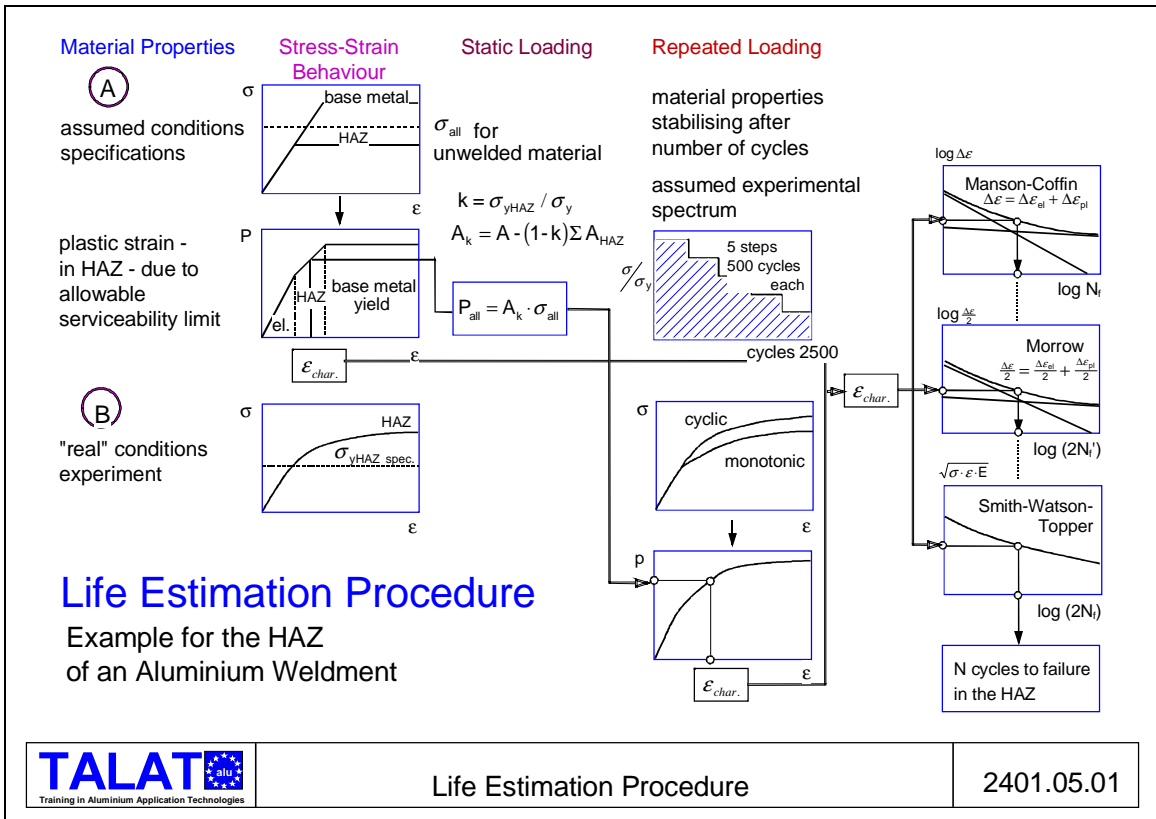
with $\sigma_a = \Delta\sigma/2$ and $\varepsilon_a = \Delta\varepsilon_T/2$

The right side of this equation enables the simple calculation of characteristic values in the form of $(\sigma_a \cdot \varepsilon_a \cdot E)^{1/2}$ vs. reversals to failure $2N_f$. The resulting curve characterises the fatigue behaviour of the material and can be readily used in design, since for design purposes the value $(\sigma_a \cdot \varepsilon_a \cdot E)^{1/2}$ is equal to $K_f \cdot S_a$

with

K_f from equation (9)
 S_a nominal stress amplitude = $\Delta\sigma/2$

A schematic diagram of the above described procedure follows in **Figure 2401.05.01**.



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- [1] Kosteas, D., Steidl G. and Strippelman W. - D.: Geschweißte Aluminium-Konstruktionen. Vieweg, Braunschweig, 1978.
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Remarks

Further proposals have been made for the application of the local notch theory concept in estimating fatigue behaviour. These proposals, especially by Seeger or Radaj, provide for different solutions to the calculation of the effective strain values at the notch.

Details for calculation of respective values are given in:

Seeger T., Beste A.:

"Zur Weiterentwicklung von Näherungsformeln für die Berechnung von Kerbspannungen im elastisch-plastischen Bereich"

VDI-Fortschrittsberichte, Vol. 18(2), pp. 1-56, 1978

Radaj D:

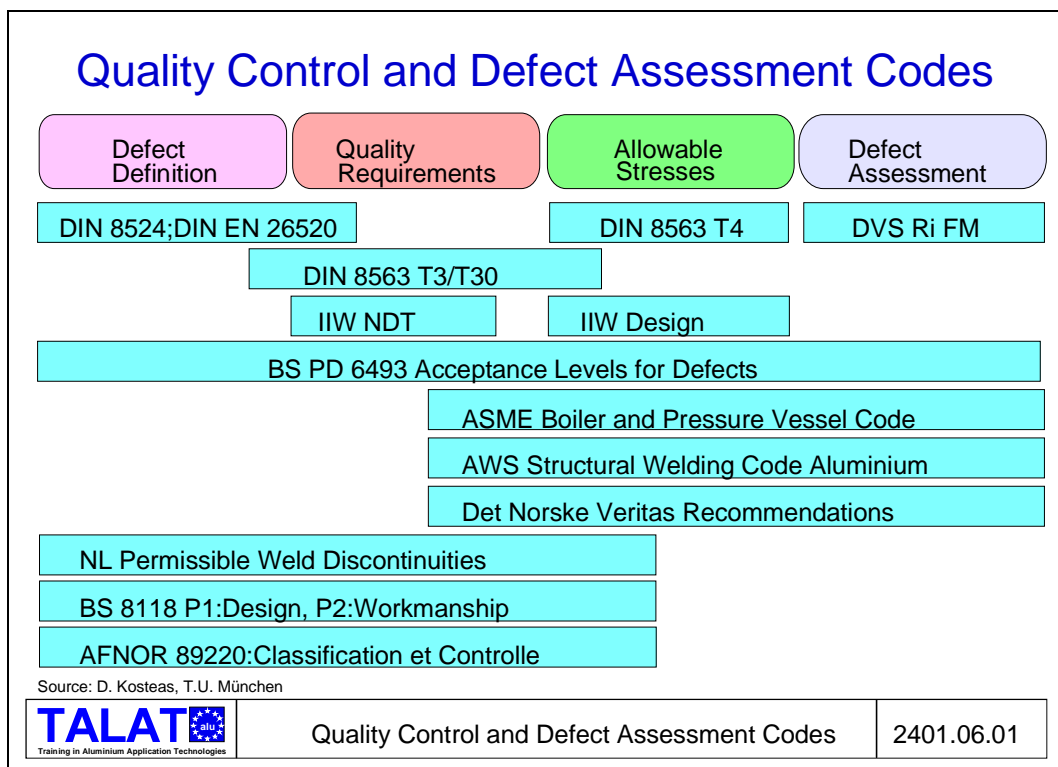
"Gestaltung und Berechnung von Schweißkonstruktionen"

Deutscher Verlag für Schweißtechnik

2401.06 Effects of Weld Imperfections on Fatigue

- Types of imperfections
- Influence of imperfections on static strength
- Influence of imperfections on fatigue strength

In the last few years codes for the definition of weld defects, weld quality and the derivation of acceptance levels for weld defects have been introduced in several countries, **Figure 2401.06.01**, covering steel as well aluminium structures. For a wide range of quality control criteria the codes for use in structural engineering are based on empirical experience and the possibilities to measure the defect size or to avoid the defect during the manufacturing. Only in some cases have quality criteria been based on the residual strength of the joint or the structure. In this chapter weld imperfections in aluminium welded joints and their influence on fatigue behaviour will be covered. A certain overlap with **Lecture 2404.02** cannot be avoided, but the latter will refer mainly to attempts of quantification of influence in relation to classification of structural joint details.

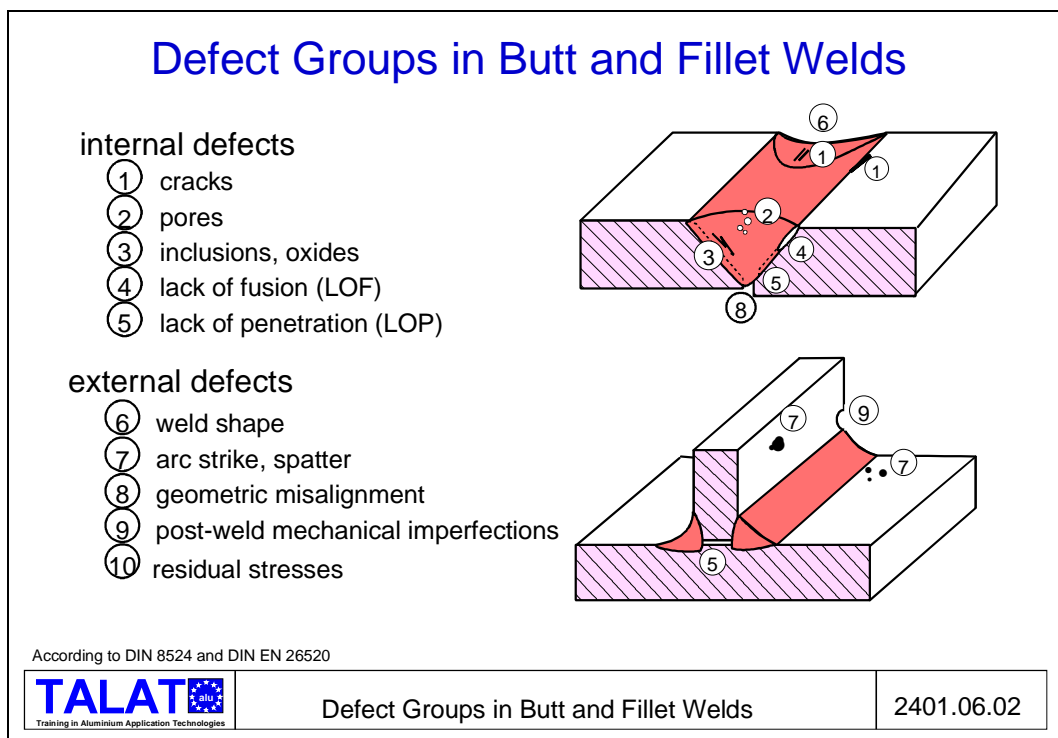


The quantification of effects of weld imperfections on the static and the fatigue behaviour of structures in different engineering applications is associated with several problems. Different loads, safety and quality control concepts complicate the evaluation. In the case of welded aluminium structures there is also a lack of data in some areas, i.e. greater cracks and cracks in the heat affected zone (HAZ), as well as aluminium specific problems, i.e. the reduced strength in the weld and in the heat affected zone.

Types of Imperfections

Weld imperfections can be classified by different characteristics, for example internal or external, source of imperfection, or the type of the imperfection. Another way is the classification based upon a fracture mechanics approach in crack-like, stress raising and insignificant imperfections. This shows that weld imperfection classification depends on the standpoint of the classifier, in general the organisation issuing the code. If quality control engineers and welding specialists dominate, the codes may be oriented more on the possibility of detection or avoidance of a weld imperfection.

A detailed description of possible weld imperfections is given in DIN 8524 and DIN EN 16520. **Figure 2401.06.02** shows the major groups of weld imperfections. The proposed German quality standard DIN 8563, T30 uses 19 single characteristics for butt welds and 14 for fillet welds, 5 of the first and 4 of the second are determined by quantitative rules. The others are described qualitatively. There are 4 classes for butt welds (AS, BS, CS, DS) and 3 for fillet welds (AK, BK, CK). These characteristics are only a part of a detailed list of 110 possible weld imperfections in DIN-EN 16520.



Cracks usually form as hot cracks in the heat affected zone or the weld itself during the cooling period. For standard alloys and filler metals problems arise generally from cracks at end craters only. They can be minimised through joint optimisation a welding plan and a welding sequence, qualified welding personnel. Alloys with higher silicon concentration may develop cracks in the remelted base material. This is a problem depending on the combination filler-base metal and the welding parameters. Greater cracks can reach up to some millimetres in length and depth. They may grow up to visible sizes after a number of load cycles. Care should be taken in welding over of greater cracks or in multilayer welds which may lead to residual internal cracks difficult to detect.

Pores result from the reduction of the volume during its solidifying process, from humidity and/or gas input into the fusion zone associated with a high welding speed. Volume reduction leads to microporosity less than 0.25 mm in diameter. Hydrogen gas in solution in the melt diffuses during solidification into micropores. These micropores may coalesce to macropores by remelting through a second weld pass. Larger pores may result from water vapour or moisture in the shielding gas or moisture on the base and/or filler metal. Another source of water vapour are defect torch cooling systems.

Inclusions larger than 0.1mm are the result of a defective cleaning of material surfaces. Slag inclusions which present a significant problems with steel welds are no problem for aluminium welds, commonly performed in MIG, TIG, electron beam or plasma techniques. Larger oxides or lack of fusion results from wrong welding parameters and/or improper removal of the initial oxide film. Oxides smaller than 0.3 mm are unavoidable.

Lack of penetration will be determined by the weld preparation geometry and the welding parameters. Fillet welds exhibit by definition lack of penetration. But a sound weld root or a double fillet weld is always beneficial.

The weld shape is controlled by the welding position, the welding parameters, especially the heat input, the qualification of the welders and the equipment. The welding personnel is also responsible for frequency of arc strikes and the volume of spatters.

Geometric misalignment and thermally induced deformations are a general and common problem for welded structures. They may affect the appearance of the structure and more significant, the strength and the function of the components. Deformations can be limited to acceptable values by carefully designed welding procedures and sequences, appropriate fixtures for the joint parts etc.

Mechanical deformations due to post weld treatments, transportation and erection are often unavoidable.

Residual stresses due to welding are commonly not discussed in the quality codes. But nevertheless they are one of the main parameters affecting fatigue behaviour of welded structures. Accounting for uncertainties in the calculation and determination of residual stresses in the hot spot areas the design codes assume the presence of residual stresses and their magnitude equal to the yield strength level in practically all welded joints. Whether residual stresses will show a relaxation or not during fatigue cycling in certain alloys or joint configurations is still an open issue. Recommendations account for a bonus factor, if a residual stress relaxation can be verified, as is the case with the ECCS Recommendations for Aluminium Alloys Structures in Fatigue, see **Lecture 2402.01**. A possibility to reduce residual stresses and simultaneously offering a beneficial geometric effect as well, is peening, TIG-dressing, stress-relieve-annealing. In this context we may also mention the specific problem of welded aluminium joints covering at least 3 different material property areas, base metal, heat affected zone and weld zone. These may be regarded as a metallurgical notch effect.

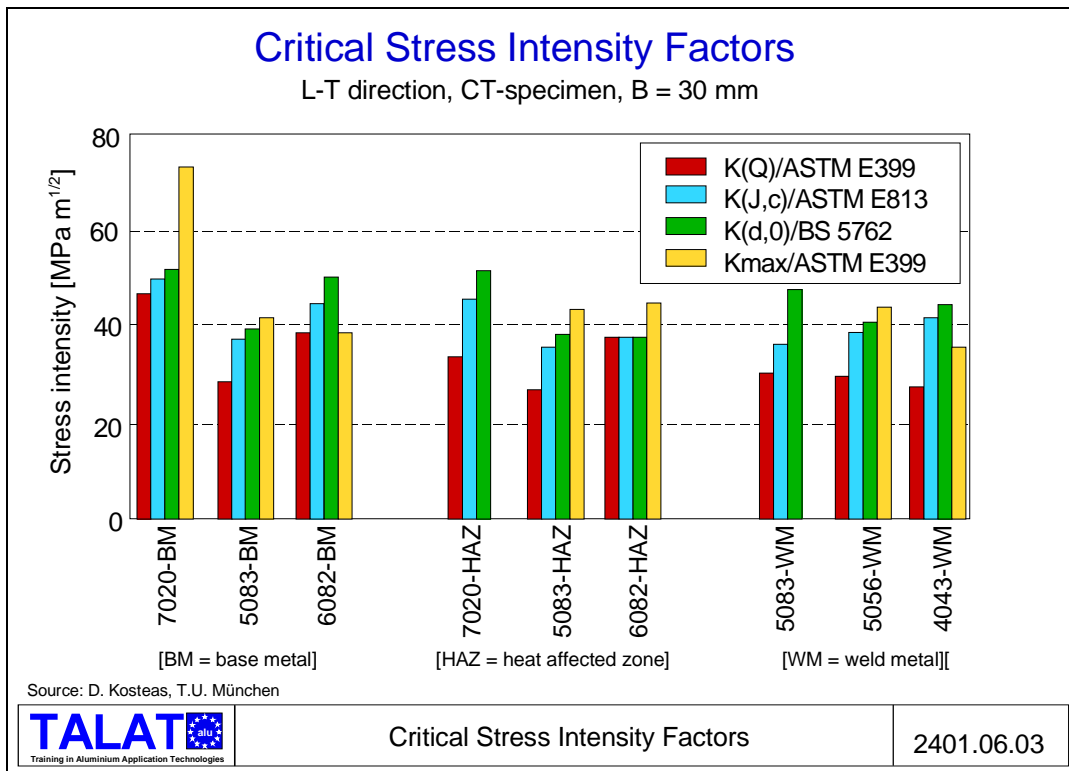
Influence of Imperfections on Static Strength

Aluminium Alloys customarily used in structural engineering like 5083, 5086, 6061, 6082, 7004, 7020 show favourable toughness and sufficient to good ductility values in the base metal and the different zones of the weldment. Problems may arise in the reduced strength of relatively large heat affected zones, especially for work hardened alloys, or in the weld itself. This leads to a concentration of a deformation of a welded joint. Stresses will be raised locally and crack like weld imperfections will further concentrate the deformation in small regions of the joint. Reduced ductility of the joint is the result which may lead to an early failure of the structure, although the latter is still under fully elastic strain. It is therefore indicated that in statically loaded structures acceptance levels for weld imperfections are a ductility problem of the joint rather than a strength problem. Tests with full scale tubular joints show a tougher behaviour for lower strength base metal compared to high strength base metal, because of a better redistribution of deformations. The maximum load capacity is not influenced by the strength of the base metal significantly.

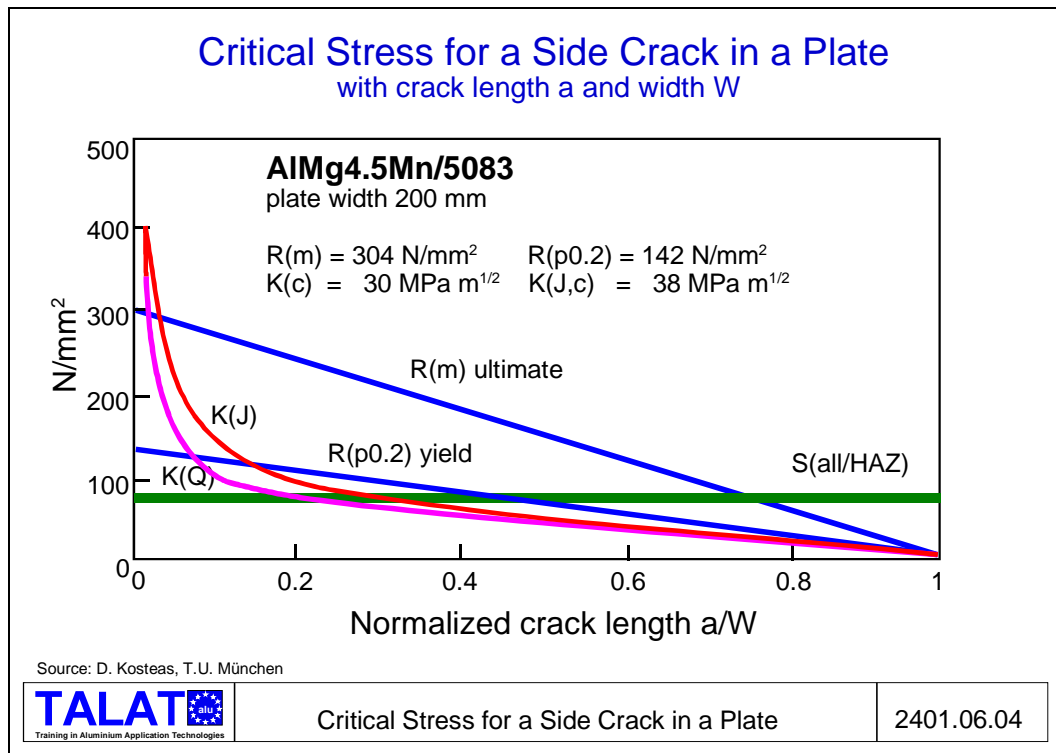
Static tests in small specimens show a similar behaviour. The strength of the welded joint is normally a function of the strength of the weld metal, as long as this is lower than the strength of the base metal or of the heat affected zone. This can be derived also from the critical stress intensity factors for the different zones of the weldment as demonstrated in the K -values measured according to different methods or recommendations, see Table 1 (**Figure 2401.06.03**):

Alloy	K_Q MPa \sqrt{m}	$K_{J,c}$ MPa \sqrt{m}	$K_{\delta 0}$ MPa \sqrt{m}	K_{max} MPa \sqrt{m}
AlZn4,5Mg1	47	50	52	73
AlMg4,5Mn	29	38	40	42
AlMgSi1	39	45	51	39
AlZn4,5Mg1-HAZ	35	46	52	-
AlMg4,5Mn-HAZ	27	36	39	44
AlMgSi1-HAZ	38	38	38	45
S-AlMg4,5Mn	30	43	47	-
S-AlMg5	29	38	40	43
S-AlSi5	26	40	43	34

Table 1: Stress Intensity Factors for Weldments, L-T Direction, CT-Specimen, B = 30mm, K_Q and K_{max} after ASTM E399, $K_{J,c}$ after ASTM E813, $K_{\delta 0}$ after BS 5762



The diagram in **Figure 2401.06.04** demonstrates that such stress intensity factors permit crack like defects of some millimetres size to be tolerated without significant reduction of strength.



Also at cryogenic temperatures brittle fracture is no problem with welded aluminium joints. Static fracture toughness as well as ultimate strength will increase with decreasing temperatures, while the impact Charpy energy values will decrease slightly with

decreasing temperature. The lower Charpy values in the welded zone compared to those in base material are a result of the lower ultimate strength respectively. Internal imperfections like porosity reduce the Charpy values only for relatively high pore density. No decrease could be detected even with fatigue cracks as starter notches up to porosity values of 15%. Porosity up to 45% reduces the impact toughness slightly by the amount of reduction of the area. This is true for the static strength values of a joint as well. All this demonstrates that statically loaded welded aluminium structures are relatively insensitive to imperfections as far as these do not reach considerable dimensions. Geometrical notches due to misalignment should be taken into account though in the calculation of stresses.

Butt welds welded from one side only, or unsatisfactory weld root form in fillet welds or the unavoidable gap in double fillet welds forms the imperfection defined as lack of penetration (LOP). Unsatisfactory connection between weld and base metal will lead to lack of fusion (LOF). LOP and LOF are two imperfections that will affect strength significantly. Especially LOP may be unavoidable in certain joint configurations and plate thicknesses because otherwise the necessary weld energy input would result in a significant heat affected zone, i.e. potential crack site. Static strength will decrease in a linear relationship with increasing imperfection size. Even in intermittent LOP it seems that the strength reduction depends on the imperfection size rather, i.e. depth of imperfection than the reduction of area due to this imperfection. One sided LOP will affect strength more severely, since eccentricity causes additional bending stresses.

Influence of Imperfections on Fatigue Strength

Fatigue life is dominated by crack propagation especially in welded joints. Cracks of approximately 100 μm are initiated during the first 10 % of total life to failure. If the measurable crack size can be reduced to approx. 10 μm this ratio may be reduced below 1% of total life. This was established for unnotched base metal as well as for notched base metal and welded joints. Similar behaviour has been observed in high strength and welded structural steel. Fatigue behaviour can, therefore, be viewed largely as a result of crack propagation behaviour and of the stress conditions in the critical zones, hence the importance of fracture mechanics.

Based on this knowledge the influence of weld imperfections can be derived through fracture mechanics calculations as defined in recommendations such as DVS-Merkblatt 'Bruchmechanische Bewertung in Schweißverbindungen', BS PD6493 'Guidance on some methods for the derivation of acceptance levels for defects in fusion welded joints', and ASME Boiler and pressure vessel code 'Analysis of flaw indication'. Crack initiation time is thereby neglected. Details on the fracture mechanics assessment are given in **Lecture 2403**.

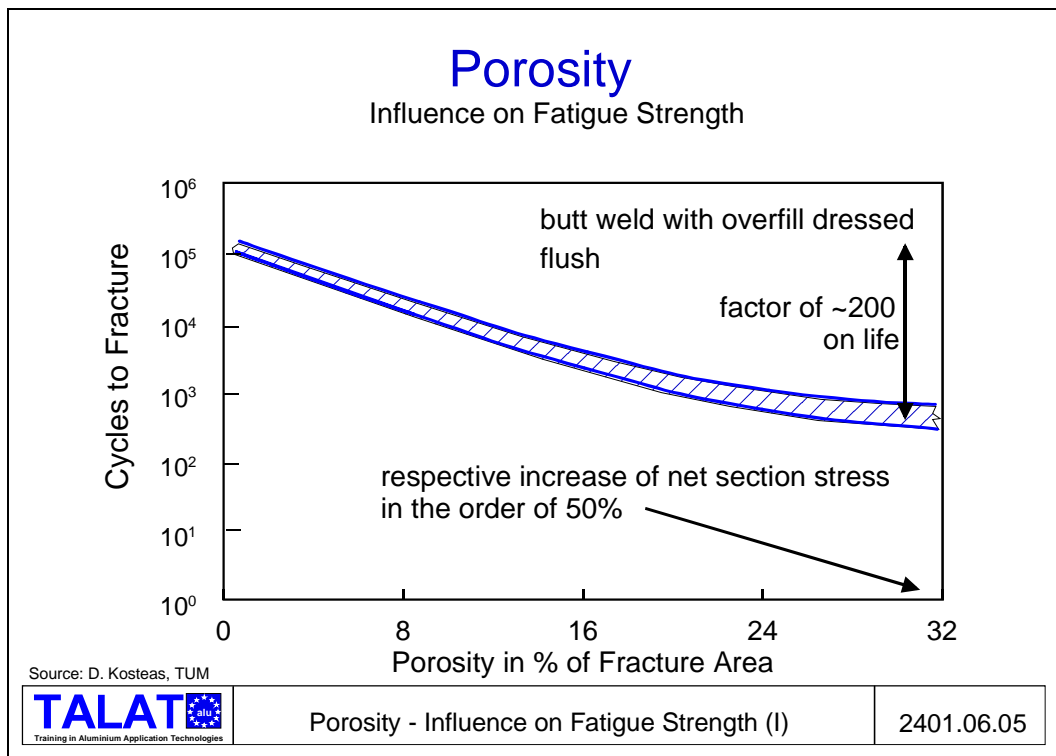
Numerous and detailed empirical investigations exist for most types of weld imperfections in welded aluminium joints. A literature documentation on imperfections is available within the Aluminium Data Bank at the Technical University of Munich and lists more than 300 publications dealing directly with weld imperfections.

Cracks

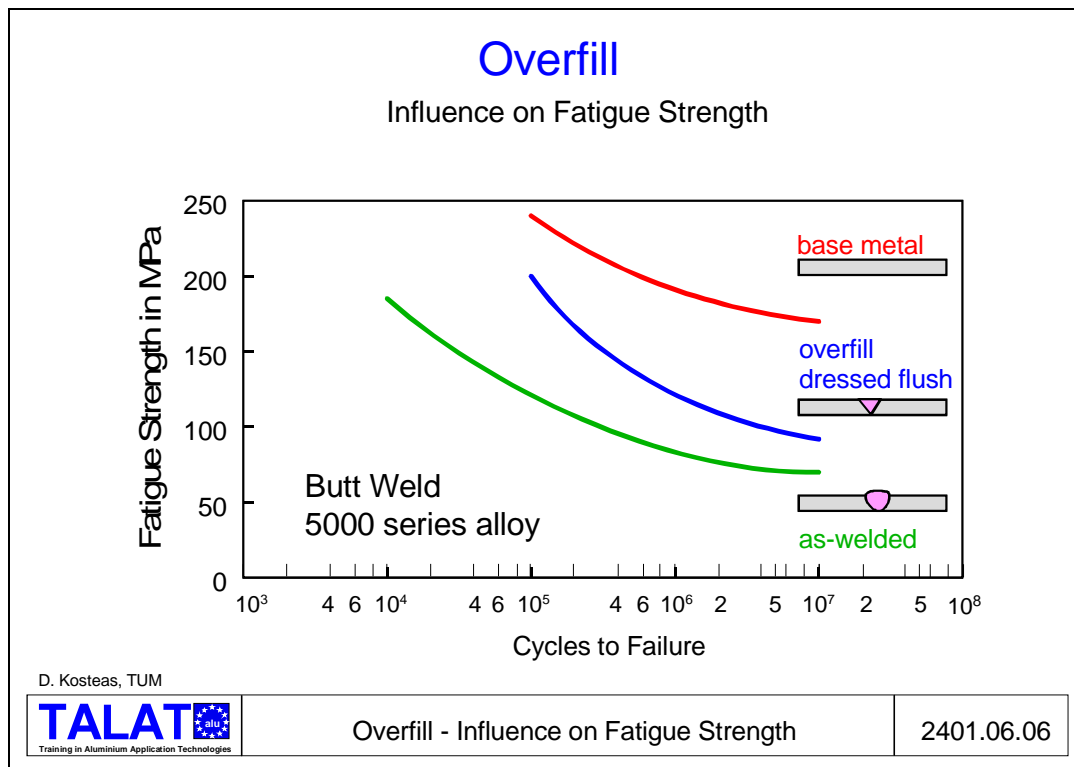
Data on the fatigue behaviour of hot or cold cracks is not so numerous. Some published data indicate a 10% reduction of the fatigue strength for rewelded crater cracks. Some data with full scale welded beam fatigue tests shows no influence of rewelded or surface crater cracks. No fractures could be detected here originating from crater cracks. This may be explained through a full remelting of possible initial cracks by the second weld pass or that in all cases where surface crater cracks had been detected, these were in the direction of the principal stress. Care should be taken though for transverse cracks which will reduce fatigue strength significantly.

Porosity

Porosity up to 35% will reduce fatigue life of butt welds with reinforcement removed (overfill ground flush) up to a factor of 200, **Figure 2401.06.05**.



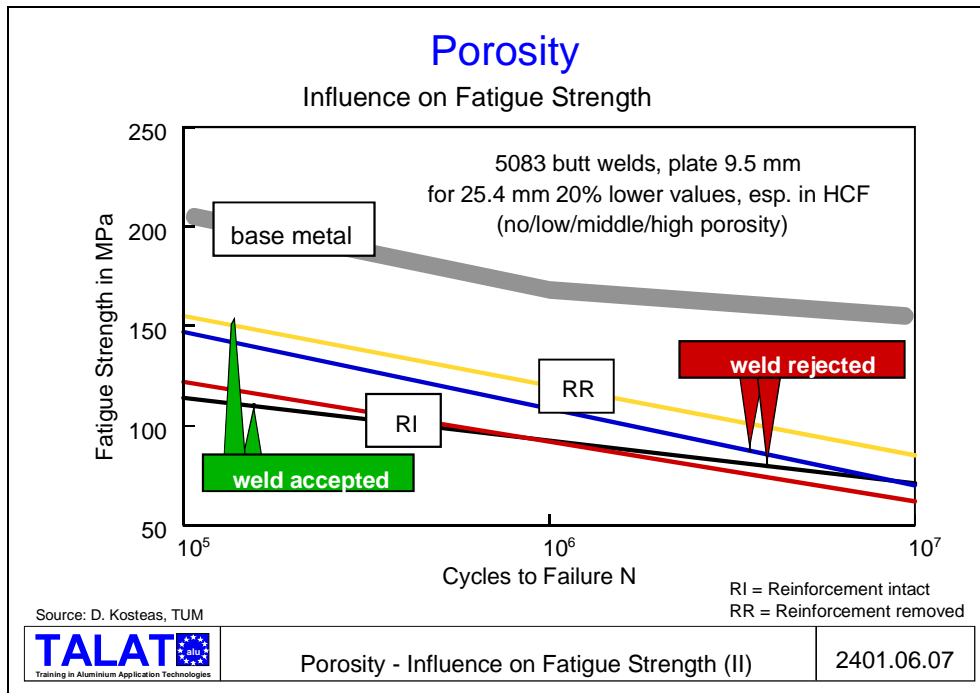
This is equivalent to a reduction in strength of 3.7 times for an S-N curve slope of $m = 4$, while the respective increase of net section stress is of the order of 50% due to the above reduction of the area. Sound as-welded butt joints with reinforcement intact show a reduction in strength of 20 - 25 % in comparison to reinforcement removed welds, see **Figure 2401.06.06**. Experimental data indicate that porosity up to certain values will not reduce fatigue strength of the as-welded joints significantly. Even values of 15% porosity have been reported as insignificant to the fatigue strength. Much will depend though on the size of individual pores, on their distribution within the weldment.



On the basis on fracture mechanics calculations and empirical data the maximum pore size has been recognised as a characteristic and more accurate parameter influencing fatigue strength. Single large pores are always more severe than the always present fine to intermediate porosity. In DVS: Merkblatt 1611 'Beurteilung von Durchstrahlungsaufnahmen im Schienenfahrzeugbau - Schmelzschweißverbindungen an Aluminium und Aluminiumlegierungen' the diameter of a single pore is limited to 30 - 43 % of the plate thickness up to a maximum value of 6.4mm. Test data and fracture mechanics calculations indicate an acceptable pore size of 1-2mm in as-welded joints and 0.2 - 0.5 mm for reinforcement removed butt welds. The fatigue strength of fillet welds will normally not be affected significantly by porosity, since the severe notch effect at the weld toe or the lack of penetration of the root overshadows the influence of porosity. As a consequence, larger single pores or higher porosity percentages may be tolerated.

Small butt welded specimens with sound reinforcements removed show porosity induced fractures only for pores near or at the surface with sizes of 0.2 - 0.5 mm. Fracture mechanics calculations show that the K_I values are higher by a factor of 1.5 for imperfections at or near the surface than for internal imperfections of the same size. Based on this the acceptable imperfection size can be increased for internal imperfections by a factor of 2.25. Secondary bending stresses increase the detrimental effects of surface imperfections. If surface imperfections are the cause of fatigue fracture, another fact must be mentioned which is the relative insensitivity of the test results with respect to stress conditions, whether axial tension or bending. On the other hand bend tests can lead to unsafe prediction or non-comparable results concerning the effects of internal imperfections, since reduced local stresses will act at the imperfection site. Welds in structures are usually stressed by axial tension and only by a small bending component. Therefore axial tension tests are more realistic in most cases.

The diagram of **Figure 2401.06.07** depicts fatigue test results of butt welds in AlMg4,5Mn with 4 different porosity densities (sound welds, low, middle, and higher porosity). Specimens with removed reinforcements show a higher fatigue strength than those with reinforcement left intact. Tests were performed on a plate thickness of 9.5 mm respective values for 25.4 mm thickness lie at approximately 20 % lower strengths especially in the high cycle region. In general porosity will not affect the behaviour of welds with reinforcement intact, only in case of a very flat weld profile and a very high pore density. Porosity will have a significant effect on fatigue strength in case of removed reinforcement especially for low fatigue strengths.



Inclusions, Oxides

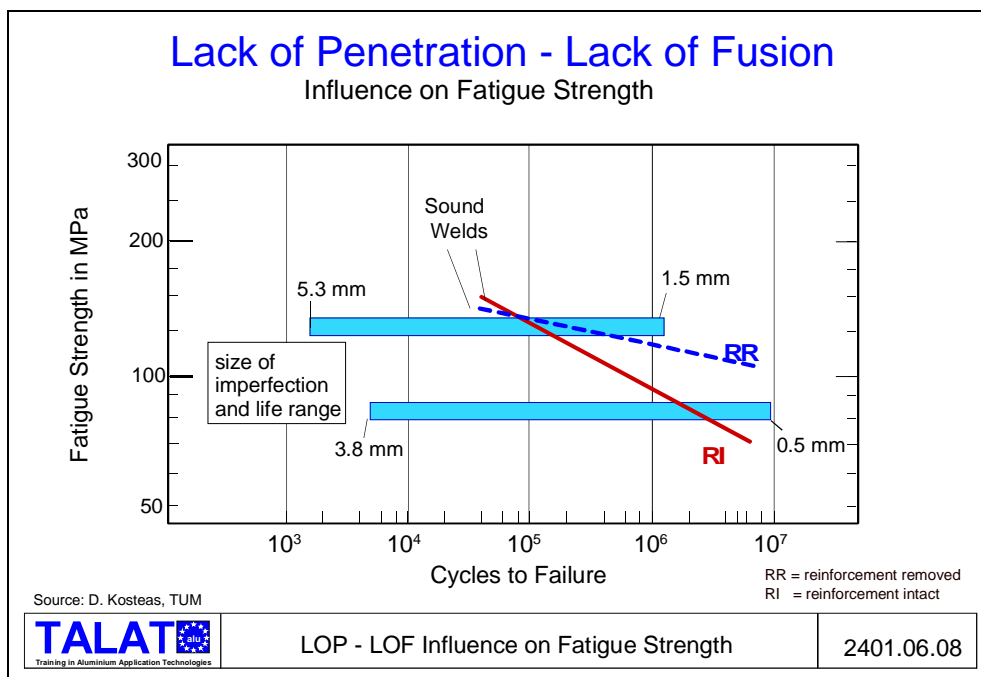
Oxides form crack-like imperfections because of their planar character with a thickness of 1 to 10µm and sizes below 0.5mm. Near-surface or surface oxide inclusions are common crack initiation sites in sound welds. This was also observed for sound as-welded butt and fillet welds. Larger oxides will reduce the fatigue strength significantly. As a result of their very small dimensions, especially thickness, oxides are very difficult to detect by non-destructive test methods.

For aluminium welds the inclusion problem is not as severe as for electroslag steel welds. Some investigations show no or very small effect of inclusions upon fatigue strength. This will be especially the case when geometric effects of the weld profile override possible notch effects of inclusions, as is often the case with fillet welds.

Lack of Penetration - Lack of Fusion

Lack of penetration (LOP) and lack of fusion (LOF) are similar imperfections but they may show different behaviour. Both will exhibit a detrimental effect on fatigue strength. The opposite surface of a LOF imperfection will usually be pressed together by residual stresses and this will lead to higher fatigue lives. On the other hand the imperfection may not be detected easily by means of non-destructive testing. Tests have showed that the length of the imperfection in the weld direction as the commonly used parameter to restrict LOP and LOF imperfections is inadequate to describe their influence on fatigue strength. The relevant parameter should rather be the width of the imperfection transverse to the maximum stress direction. This can be derived from fracture mechanics considerations as well. This imperfection size will usually be the through-thickness imperfection width, for which identification problems by non-destructive testing are in common. Using ultrasonic inspection methods one has to use special transducers for diagonal testing. X-ray tests record only the perpendicular projection which may underestimate the value significantly.

Acceptable imperfection sizes can be derived from test results, **Figure 2401.06.08**. For as-welded joints the LOP size may be between 1 mm and 2 mm, this is compatible with fracture mechanics analysis results. For butt joints with reinforcement removed this limit value has to be reduced below 0.5 mm for sound welds.



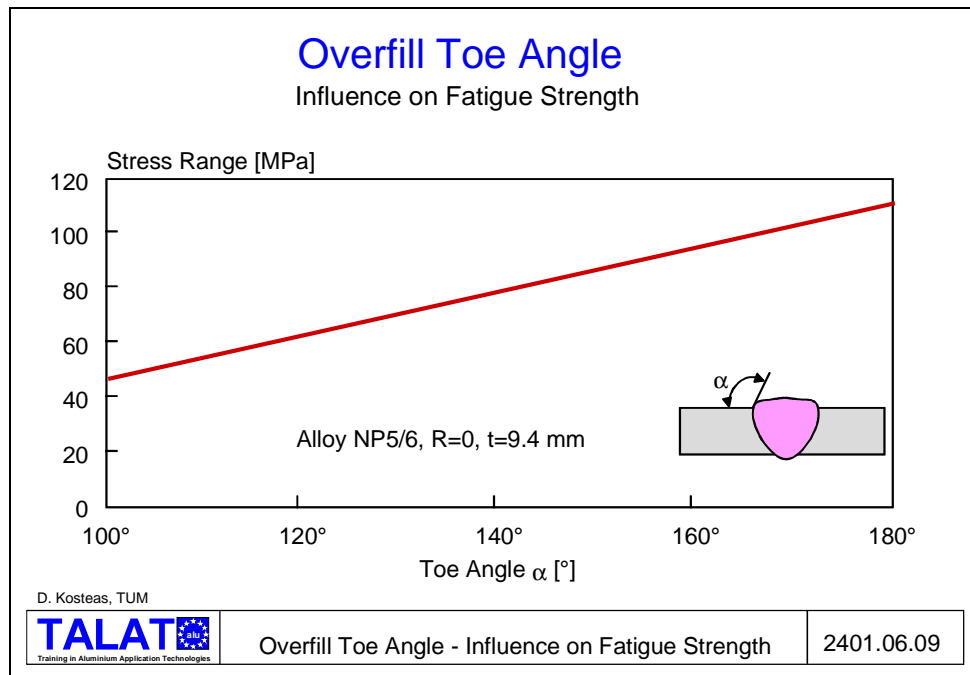
Load carrying fillet welds in cruciform joints show by definition a LOP imperfection. The fatigue strength of cruciform joints is a function of the stresses in the weld and the geometric dimensions. Test results show that, for fractures emanating from the root, the strength is directly proportional to the reduction of net section stress in the weld. For double fillet welds and a nominal weld thickness $a \leq 0.6t$ fracture will emanate from the root, a result which is also supported by fracture mechanics analysis. The transition between weld root and weld toe initiation lies in the region of $0.6 < a/t < 0.8$. It is significantly affected by the actual penetration of the fillet weld, which increases the real weld thickness. In cases of double fillet welds with preparation of joint surfaces butt like welds will result. Non-load-carrying fillet welds will usually fracture from the toe, here LOP and LOF do not affect the fatigue behaviour significantly.

Weld Shape

Major parameters of the weld shape are the reinforcement angle and the toe transition radius. The bead height is a secondary parameter. Nevertheless for butt welds the bead height and for fillet welds the convexity is the standard parameter for characterising the weld profile in recommendations except DIN 8563, T30. On the condition that the weld profile is circular with radius r the toe angle α is a function of bead height h and weld width b .

$$h = r \cdot (1 - \cos \alpha) = b \cdot (1 - \cos \alpha) / \sin \alpha \quad (1)$$

As a result the relation h/b can be used as a characteristic parameter, b depends on the weld form, V or X, thickness and pass number. The relation h/t provides a less accurate correlation. Standard values for the reduction in fatigue strength from milled to as-welded butt joints are 1.2 - 1.6, in the case of sound welds, see **Figure 2401.06.06**.



Depending on the reinforcement angle the fatigue strength at $3 \cdot 10^6$ cycles ranges from 50-110 MPa for AlMgMn welds, see **Figure 2401.06.09**. Respective values for welds with removed reinforcement lie in the range of 90-110 MPa. Based on this data an angle of up to 150° is acceptable for a strength of 90 MPa. This is equivalent to $h = 1.25 \cdot b$ and with $b = 1.4 \cdot t$ ($\beta = 70^\circ$, V-weld), we have $h = 1.37 \cdot t$.

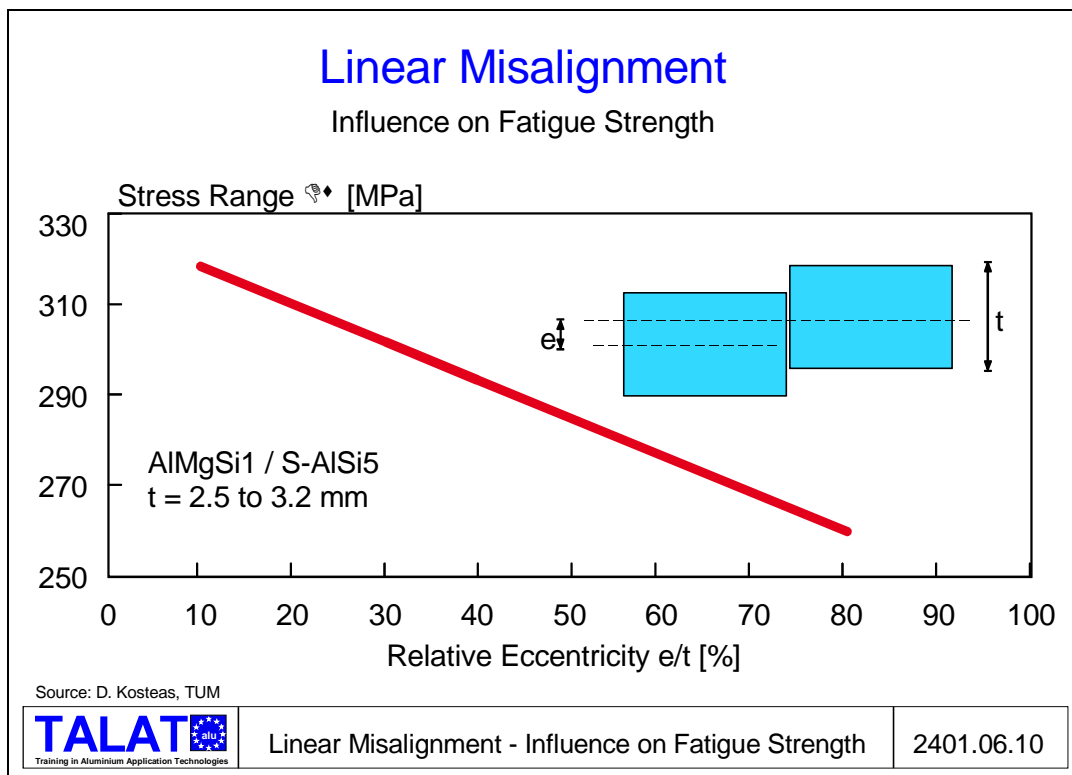
Geometric Misalignment

Geometric, linear and/or angular misalignment act as stress raisers, see **Figure 2401.06.10**. The magnitude of the secondary stress amplitude depends on the overall

design of the joint. The secondary bending stress σ_M in the case of linear misalignment of welded plates can be estimated by the simple relation

$$\sigma_M = \sigma_N \cdot 3 \cdot e/t \quad (2)$$

with σ_N denoting the axial stress, t the plate thickness and e the eccentricity. Detailed formulas are given in BS PD6493 also in the case of angular misalignment. For spectrum or block loaded structures with a certain number of cycles and stresses above the yield strength (the latter may lie below 120 MPa in the weld or heat affected zone) the angular misalignment will be reduced through plastic deformations during the first cycles.



Arc Strike, Spatter

Arc strikes outside the weld are not common in aluminium weldments. They may reduce the fatigue strength of the joint in a way similar to the reduction due to a butt weld. Spatters do not reduce the fatigue strength of aluminium welded joints.

Post-Weld Mechanical Imperfections

Post-weld mechanical imperfections may reduce the fatigue strength of reinforcement removed butt welds. Other weldtype are not affected by usual mechanical imperfections as hammer indentations or grinding notches of minor depth. Drilled holes show a fatigue

behaviour similar to sound as-welded butt welds, if net section stresses are compared. Therefore improperly drilled holes should not be repaired by filling the hole with weld material. The result may be LOP and LOF imperfections with consequent lower fatigue strengths.

2401.07 Literature/References

Further details and information on the subjects treated in these lectures may be found in the following literature

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2401.08 List of Figures

Figure Nr.	Figure Title (Overhead)
2401.01.01	Fatigue - Where?
2401.01.02	Fatigue - Location
2401.01.03	Fatigue - When and What?
2401.01.04	Fatigue - The Remedy
2401.01.05	Fatigue - Significance for Aluminium
2401.01.06	Definitions: Constant Amplitude Loading
2401.01.07	Definitions: Spectrum Loading
2401.02.01	Stress-Strain Loops: (a) Stress Control, (b) Strain Control
2401.02.02	Cyclic Stress-Strain Curve
2401.02.03	Notch-Peak Geometry at Slip Bands
2401.02.04	Fatigue Crack Initiation at a Surface Inclusion
2401.02.05	Stage I and II Fatigue Crack Growth
2401.02.06	Schematic Illustration of a Fatigue Fracture Surface
2401.02.07	Fatigue Crack Growth Rate
2401.02.08	Fatigue Crack Growth by the Plastic Blunting Mechanism
2401.02.09	Effect of Variable Load Sequence on Fatigue Crack Growth
2401.02.10	Fatigue Damage and Influencing Parameters
2401.02.11	Strain-Life Relationship
2401.02.12	Outline of Fatigue Life Prediction Methods
2401.03.01	Statistical / Regression Analysis
2401.03.02	Fatigue Test Data Analysis
2401.03.03	Statistical Methods for Calculations of Distributions and Significance
2401.03.04	Probability of Survival Lines for a Fillet Weld
2401.03.05	Probit-Method
2401.03.06	Staircase-Method
2401.03.07	Two-Point-Method (Little)
2401.03.08	Fatigue Strength Diagrams
2401.03.09	P-S-N-Curve
2401.03.10	Maximum-Likelihood-Method
2401.04.01	Cycle Counting Methods
2401.04.02	Stress-Time Diagram and Stress Spectrum
2401.04.03	Normalized Universal Spectra
2401.04.04	The „Rain-Flow“ Method
2401.04.05	Determination of Stress Spectra - „Reservoir“ Cycle Counting Method
2401.04.06	Influence of Stress Spectrum on Fatigue Strength
2401.04.07	Fatigue Behaviour Diagram
2401.04.08	Fatigue Strength for Constant and Variable Amplitude (Normal)Loads
2401.04.09	Fatigue Life Estimation
2401.04.10	Palmgren-Miner Rule of Linear Damage Accumulation
2401.04.11	Shift in Partial Damage

2401.04.12	Life Estimation with the Palmgren-Miner Rule
2401.04.13	Observed Tendencies with the Palmgren-Miner Rule
2401.04.14	Experience with the Palmgren-Miner Rule
2401.04.15	Elements of the Constant Amplitude S-N Curve
2401.04.16	Spectrum Transformation
2401.05.01	Life Estimation Procedure
2401.06.01	Quality Control and Defect Assessment Codes
2401.06.02	Defect Groups in Butt and Fillet Welds
2401.06.03	Critical Stress Intensity Factors
2401.06.04	Critical Stress for a Side Crack in a Plate
2401.06.05	Porosity - Influence on Fatigue Strength (I)
2401.06.06	Overfill - Influence on Fatigue Strength
2401.06.07	Porosity - Influence on Fatigue Strength (II)
2401.06.08	LOP - LOF Influence on Fatigue Strength
2401.06.09	Overfill Toe Angle - Influence on Fatigue Strength
2401.06.10	Linear Misalignment - Influence on Fatigue Strength